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Analytical Tool-Supported Modeling of Streaming and Stencil Loops

Georg Hager, Julian Hammer
Erlangen Regional Computing Center (RRZE)

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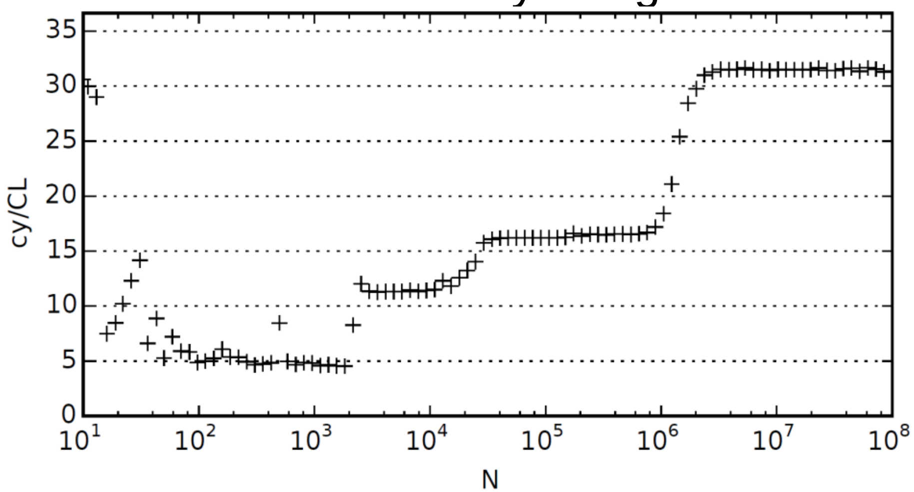
- LIKWID
  tiny.cc/LIKWID
- GHOST
  tiny.cc/GHOST
- Performance Engineering
  http://blogs.fau.de/...
      hager/talks/nlpe

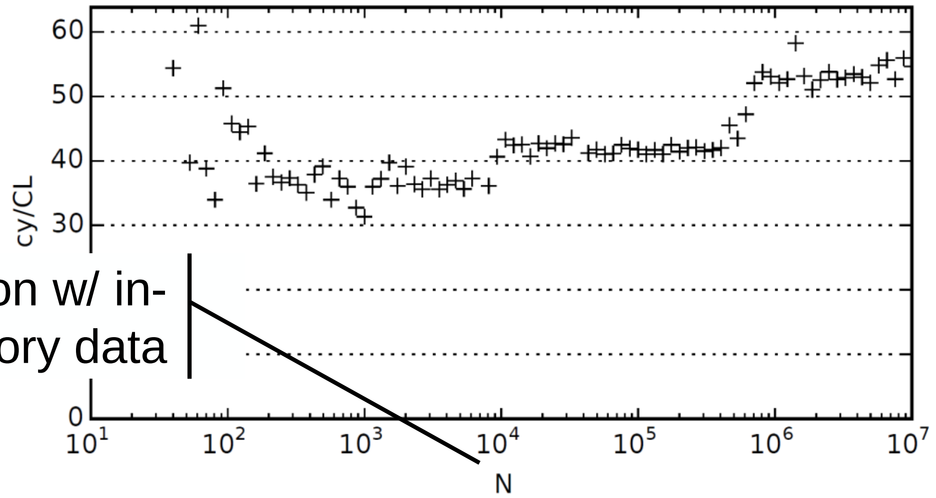
Motivation

DAXPY on Sandy Bridge core



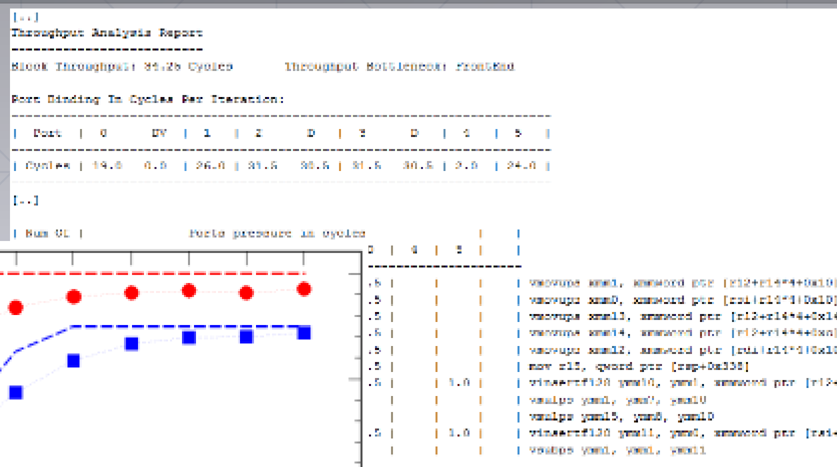
Loop length

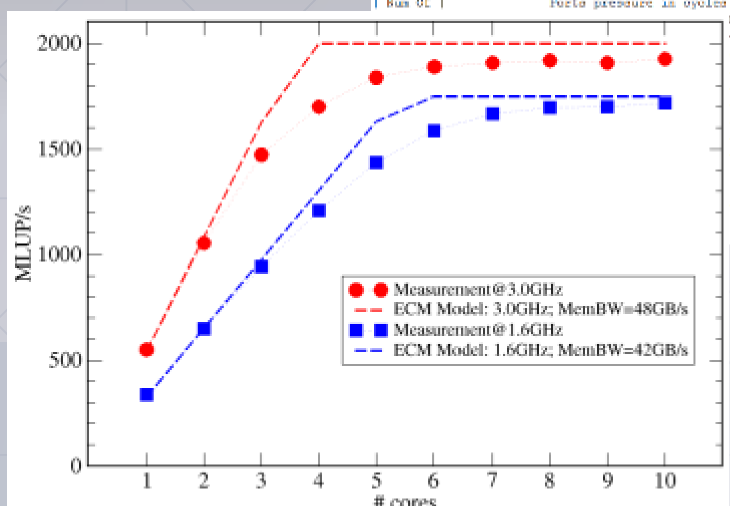
2D-5pt stencil on Sandy Bridge core

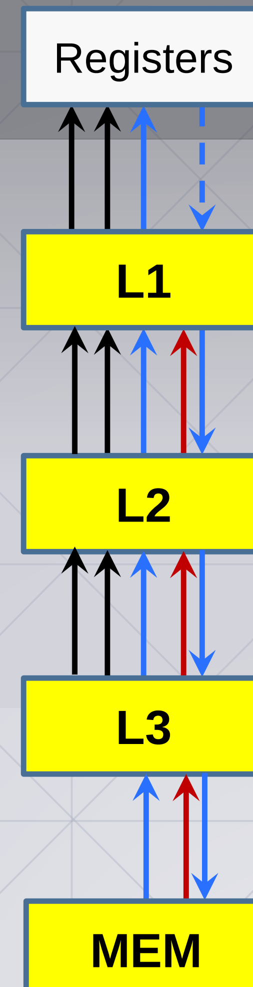


Inner dimension w/ in-memory data

THE ECM MODEL







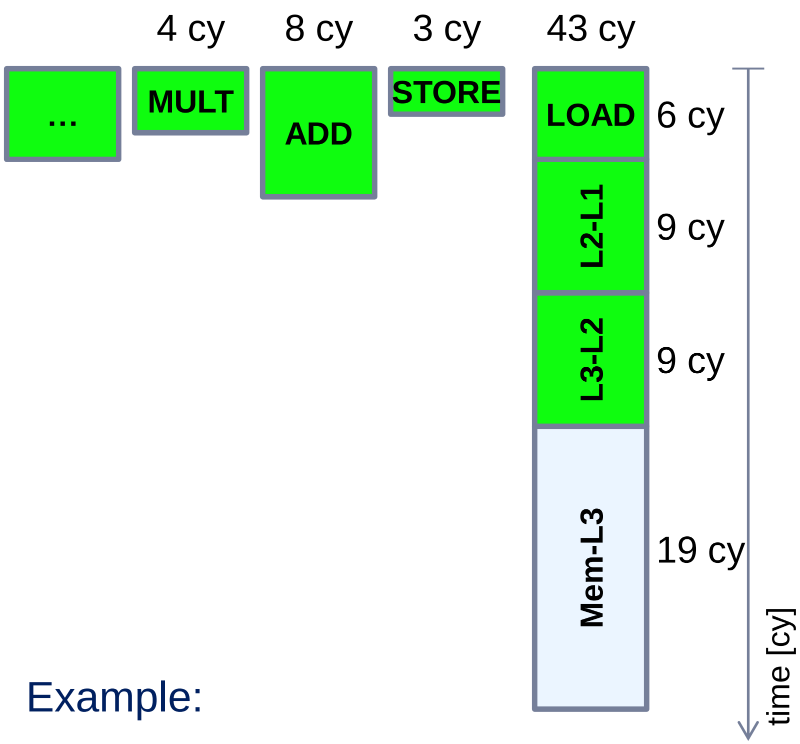
ECM model – the rules

1. LOADs in the L1 cache do not overlap with any other data transfer in the memory hierarchy

2. Everything else in the core overlaps perfectly with data transfers (STOREs show some non-overlap)

3. The scaling limit is set by the ratio of

$$\frac{\text{\# cycles per CL overall}}{\text{\# cycles per CL at the bottleneck}}$$



Example:

Single-core (data in L1): 8 cy (ADD)

Single-core (data in memory):
6+9+9+19 cy = 43 cy

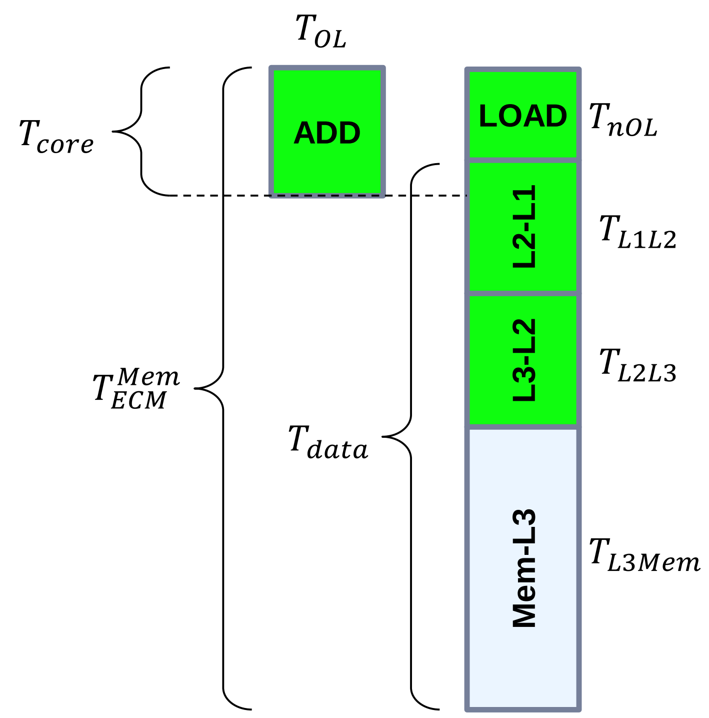
Scaling limit: 43 / 19 = 2.3 cores

ECM model – composition

ECM predicted time
T_{ECM} = maximum of overlapping time and sum of all other contributions

$$T_{core} = \max(T_{nOL}, T_{OL})$$

$$\boxed{T_{ECM} = \max(T_{nOL} + T_{data}, T_{OL})}$$



Shorthand notation for time contributions:

$$\{\, T_{OL} \parallel T_{nOL} \mid T_{L1L2} \mid T_{L2L3} \mid T_{L3Mem} \,\}$$

cy invariant to clock speed

cy changes w/ clock speed

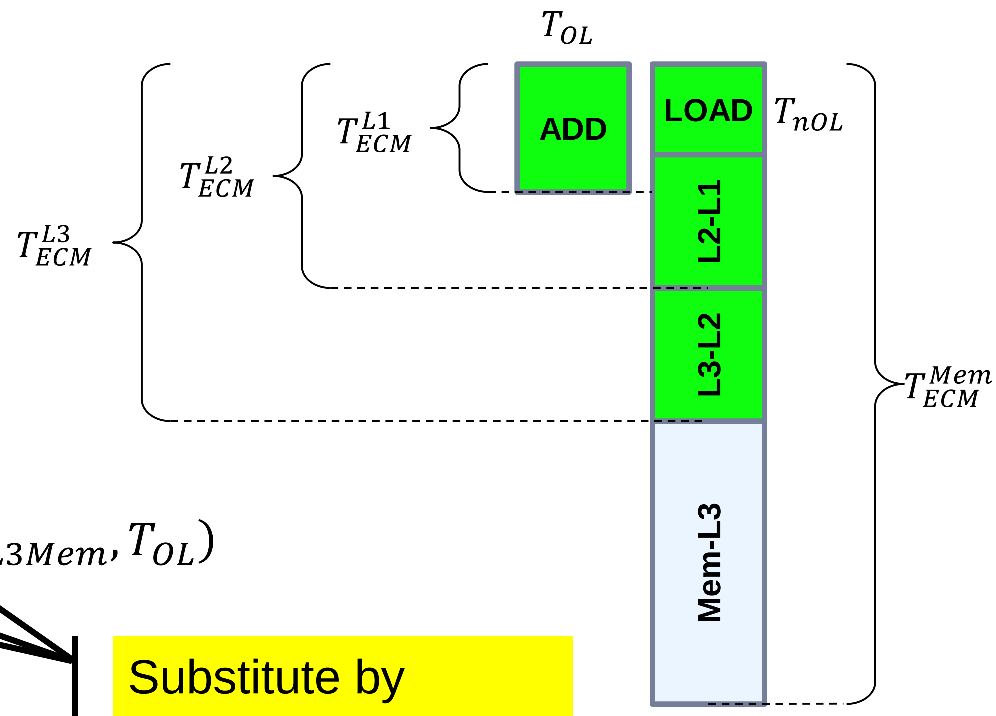
Example from previous slide:

$$\{\, 8 \parallel 6 \mid 9 \mid 9 \mid 19 \,\}\ \text{cy}$$

ECM model – prediction

Notation for cycle predictions in different memory hierarchy levels:

$$\{\, T_{ECM}^{L1} \rceil T_{ECM}^{L2} \rceil T_{ECM}^{L3} \rceil T_{ECM}^{Mem} \,\}$$



$$T_{ECM}^{L1} = T_{core} = \max(T_{nOL}, T_{OL})$$

$$T_{ECM}^{L2} = \max(T_{nOL} + T_{L1L2}, T_{OL})$$

$$T_{ECM}^{L3} = \max(T_{nOL} + T_{L1L2} + T_{L2L3}, T_{OL})$$

$$T_{ECM}^{Mem} = \max(T_{nOL} + T_{L1L2} + T_{L2L3} + T_{L3Mem}, T_{OL})$$

Example: $\{\, 8 \rceil 15 \rceil 24 \rceil 43 \,\}$ cy

Substitute by commas → Roofline

Experimental data (measured) notation: $8.6 \rceil 16.2 \rceil 26 \rceil 47$ cy

ECM model – saturation

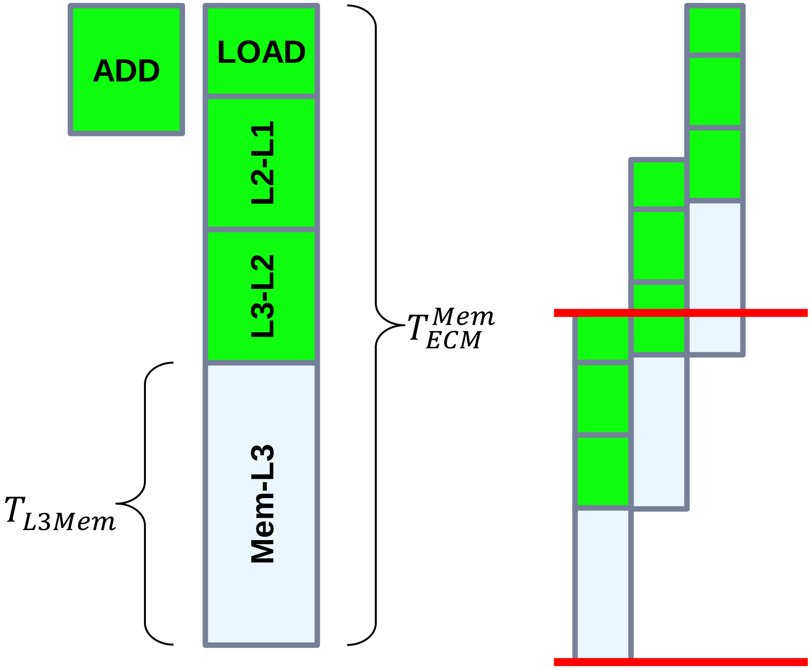
Main assumption: Performance scaling is linear until a bandwidth bottleneck (b_S) is hit

Performance vs. cores (Memory BN):

$$P_{ECM}(n) = \min\left(nP_{ECM}^{Mem}, \frac{b_S^{Mem}}{B_C^{Mem}}\right)$$

Number of cores at saturation:

$$n_S = \left\lceil \frac{b_S/B_C}{P_{ECM}^{Mem}} \right\rceil = \left\lceil \frac{T_{ECM}^{Mem}}{T_{L3Mem}} \right\rceil$$



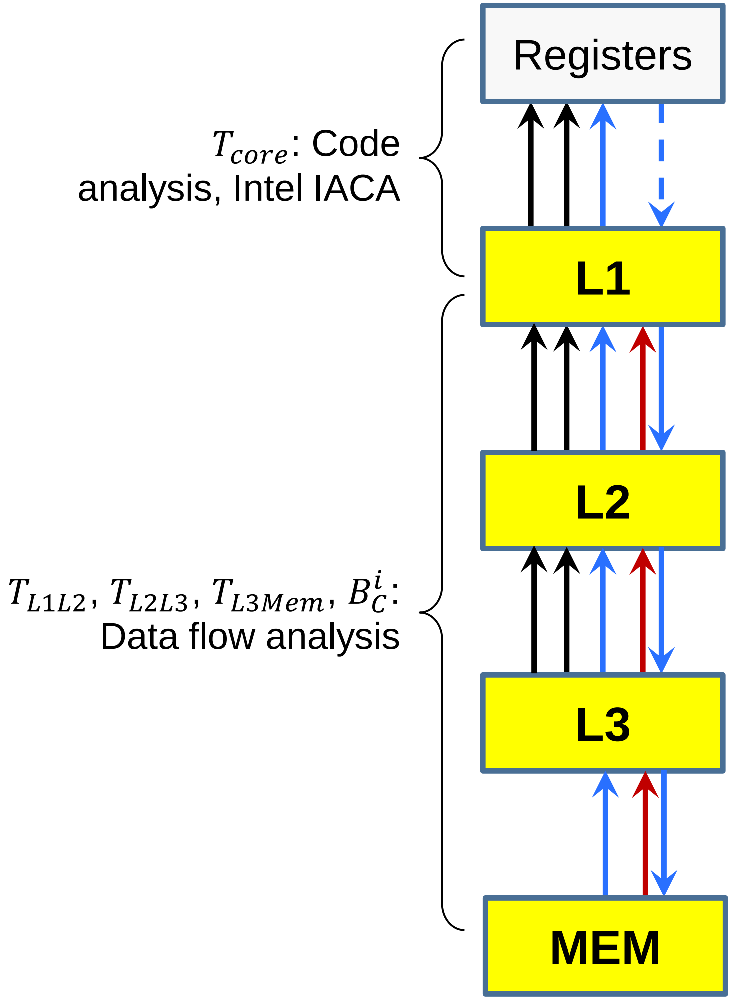
Example:

$$\{\, 8 \,\|\, 6 \mid 9 \mid 9 \mid 19 \,\}\ \text{cy}, \qquad \{\, 8 \rceil 15 \rceil 24 \rceil 43 \,\}\ \text{cy} \implies n_S = \left\lceil \frac{43}{19} \right\rceil = 3$$

How do we get the numbers?

Basic information about hardware capabilities:

- In-core limitations
  - Throughput limits:µops, LD/ST, ADD/MULT per cycle
  - Pipeline depths
- Cache hierarchy
  - **ECM**: Cycles per CL transfer
  - **RL**: measured max bandwidths for all cache levels, core counts
- Memory interface
  - **ECM**: measured saturated BW
  - **RL**: measured max bandwidths for all core counts



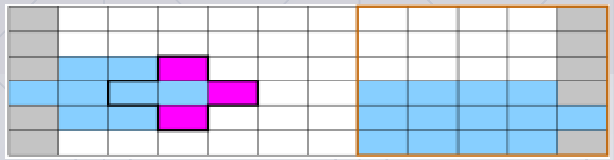
2D 5-PT JACOBI STENCIL (DOUBLE PRECISION)

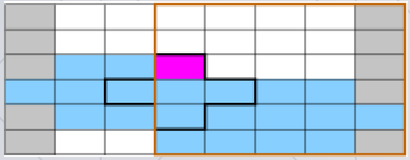
```
for(j=1; j < Nj-1; ++j)
 for(i=1; i < Ni-1; ++i)
  b[j][i] = (a[ j ][i-1] + a[ j ][i+1]
           + a[j-1][ i ] + a[j+1][ i ] ) * s;
```

Unit of work (1 CL): 8 LUPs

Data transfer per unit:

- 5 CL if layer condition violated in higher cache level
- 3 CL if layer condition satisfied





ECM Model for 2D Jacobi (AVX) on SNB 2.7 GHz

Radius-r stencil → $(2r+1)$ layers have to fit

```
for(j=1; j < Nj-1; ++j)
 for(i=1; i < Ni-1; ++i)
  b[j][i] = (a[ j ][i-1] + a[ j ][i+1]
           + a[j-1][ i ] + a[j+1][ i ] ) * s;
```

Cache k has size C_k

Layer condition:

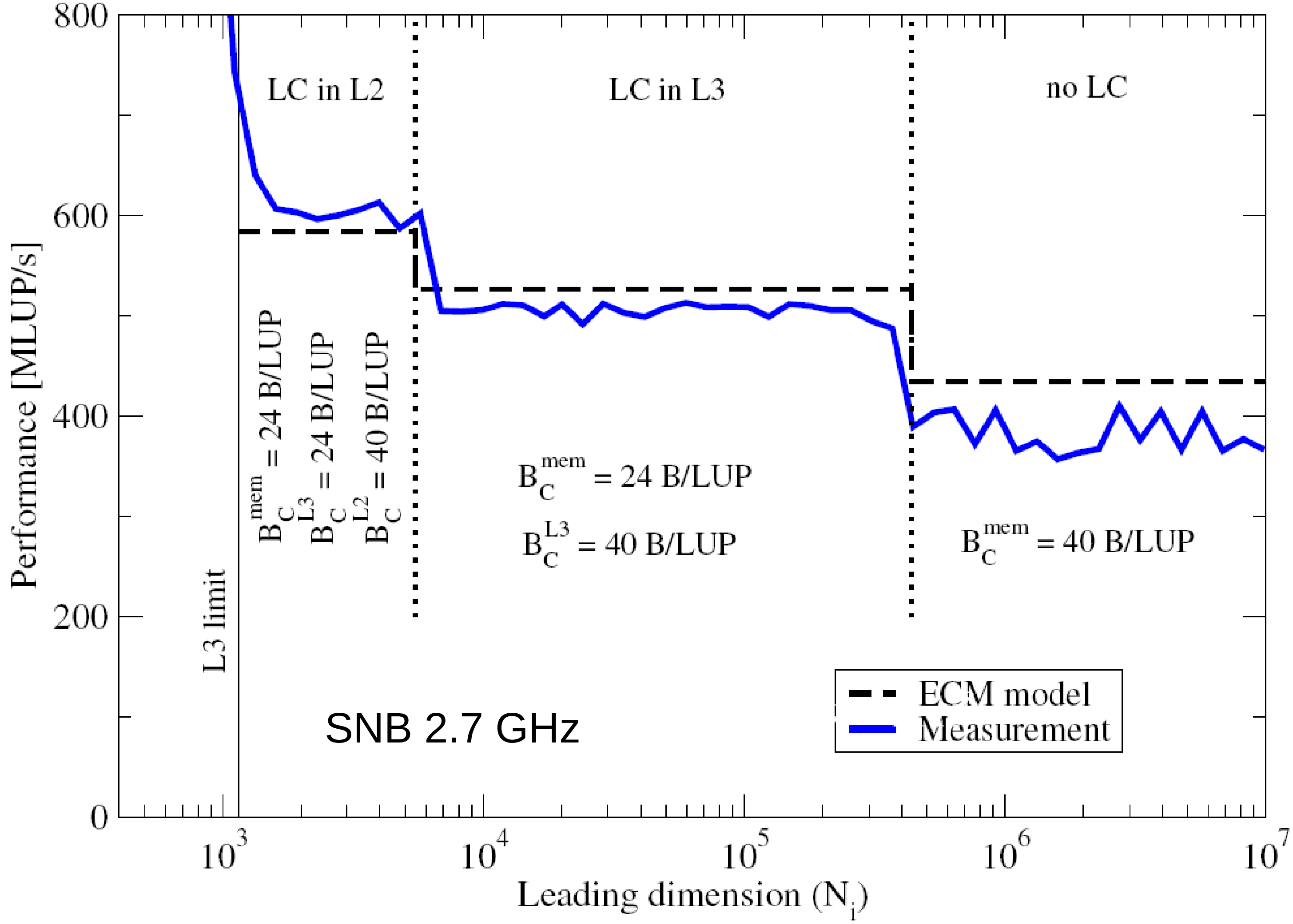
$$(2r+1)\cdot N_i \cdot 8\,\mathrm{B} < \frac{C_k}{2}$$

2D 5-pt: $r = 1$

LC	ECM Model [cy]	prediction [cy]	$P^{\mathrm{mem}}_{\mathrm{ECM}}$ [MLUPS]	$N_i <$	n_S
L1	$\{6 \,\Vert\, 8 \mid 6 \mid 6 \mid 13\}$	$\{8 \rceil 14 \rceil 20 \rceil 33\}$	659	683	3
L2	$\{6 \,\Vert\, 8 \mid 10 \mid 6 \mid 13\}$	$\{8 \rceil 18 \rceil 24 \rceil 37\}$	587	5461	3
L3	$\{6 \,\Vert\, 8 \mid 10 \mid 10 \mid 13\}$	$\{8 \rceil 18 \rceil 28 \rceil 41\}$	529	436900	4
—	$\{6 \,\Vert\, 8 \mid 10 \mid 10 \mid 22\}$	$\{8 \rceil 18 \rceil 28 \rceil 50\}$	438	N/A	3

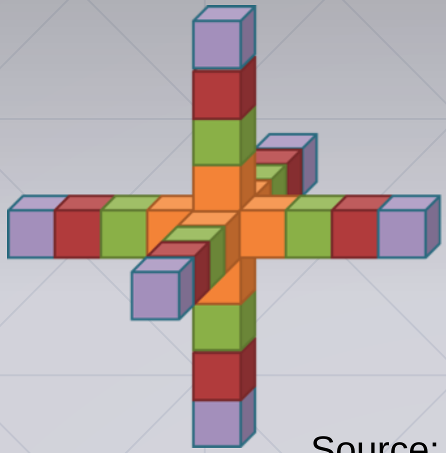
LC = layer condition satisfied in …

2D 5-pt serial in-memory performance and layer conditions



3D LONG-RANGE STENCIL (SINGLE PRECISION)

```
#pragma omp parallel for
for(int k=4; k < N-4; k++) {
 for(int j=4; j < N-4; j++) {
  for(int i=4; i < N-4; i++) {
   float lap = c0 * %V%[k][j][i]
  + c1 * ( V[ k ][ j ][i+1]+ V[ k ][ j ][i-1])
  + c1 * ( V[ k ][j+1][ i ]+ V[ k ][j-1][ i ])
  + c1 * ( V[k+1][ j ][ i ]+ V[k-1][ j ][ i ])
    ...
  + c4 * ( V[ k ][ j ][i+4]+ V[ k ][ j ][i-4])
  + c4 * ( V[ k ][j+4][ i ]+ V[ k ][j-4][ i ])
  + c4 * ( V[k+4][ j ][ i ]+ V[k-4][ j ][ i ]);
   U[k][j][i] = 2.f * V[k][j][i] - U[k][j][i]
                + ROC[k][j][i] * lap;
}}}
```



Source: http://goo.gl/dqOlnl

3D long-range SP stencil ECM model

Layer condition in L3 at problem size $N_i \times N_j \times N_k$:

$$9 \cdot N_i \cdot b_j \cdot n_{threads} \cdot 4\,\mathrm{B} < \frac{C_3}{2}$$

ECM Model: $\{\,68 \,\|\, 62 \mid 24 \mid 24 \mid 17\,\}$ cy → $\{\,68 \rceil 86 \rceil 110 \rceil 127\,\}$ cy

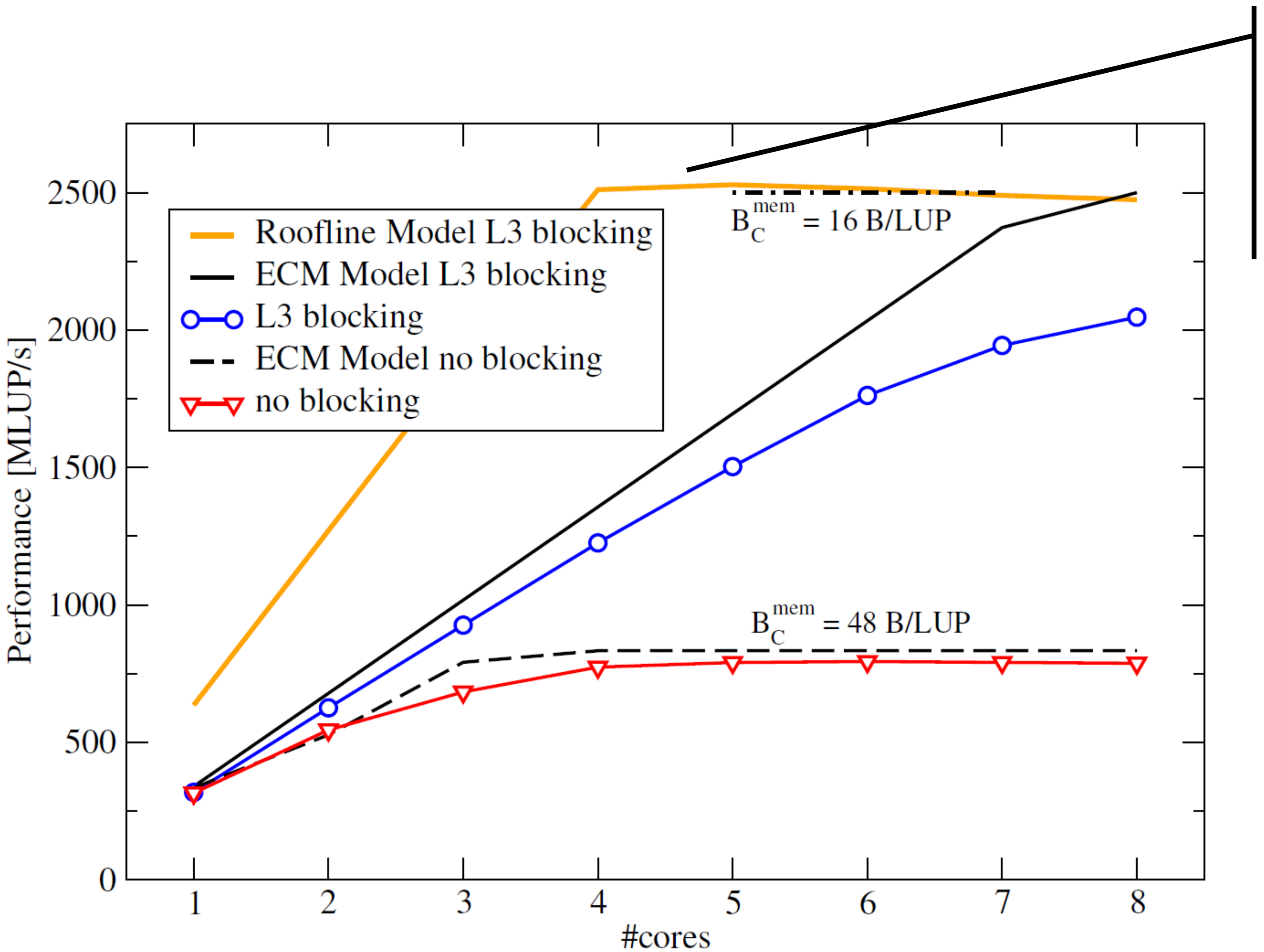
T_{L3Mem} plays minor part

Saturation at $n_s = \left\lceil \frac{127}{17} \right\rceil = 8$ cores.

Consequences:

- Temporal blocking will not yield substantial speedup
- Improve low-level code first (semi-stencil…?)

3D long-range SP stencil results (SNB)



Roofline too optimistic due to overlapping assumption

KERNCRAFT

First steps towards automated model construction

kerncraft: ECM/Roofline modeling toolkit

GitHub, Inc. [US] https://github.com/cod3monk/kerncraft

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cod3monk / **kerncraft** — Watch 1

kerncraft

Loop Kernel Analysis and Performance Modeling Toolkit

This tool allows automatic analysis of loop kernels using the Execution Cache Memory (ECM) model, the Roofline model and actual benchmarks. kerncraft provides a framework to investigate the data reuse and cache requirements by static code analysis. In combination with the Intel IACA tool kerncraft can give a good overview of both in-core and memory bottlenecks and use that data to apply performance models.

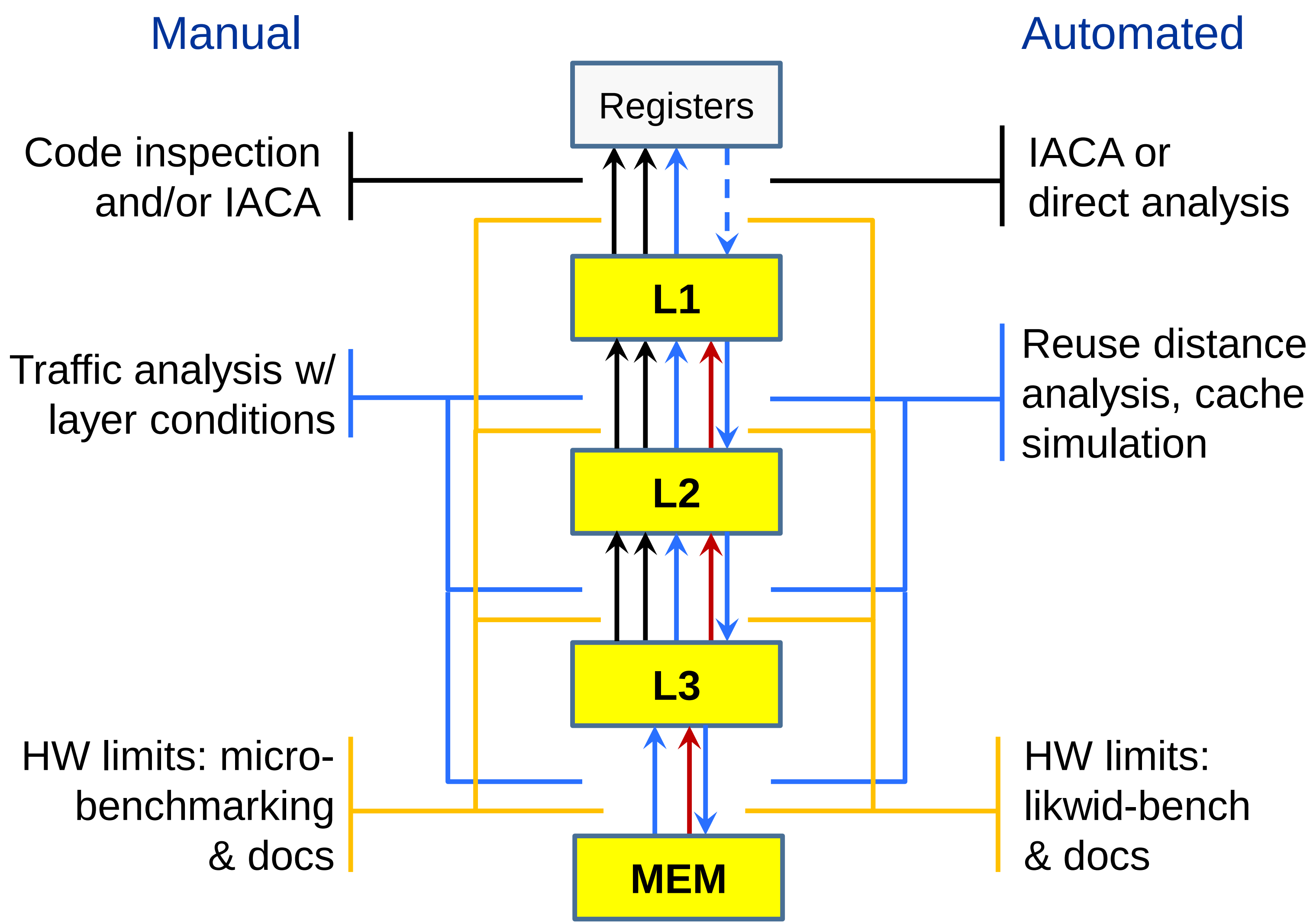
Installation

Run: `pip install kerncraft`

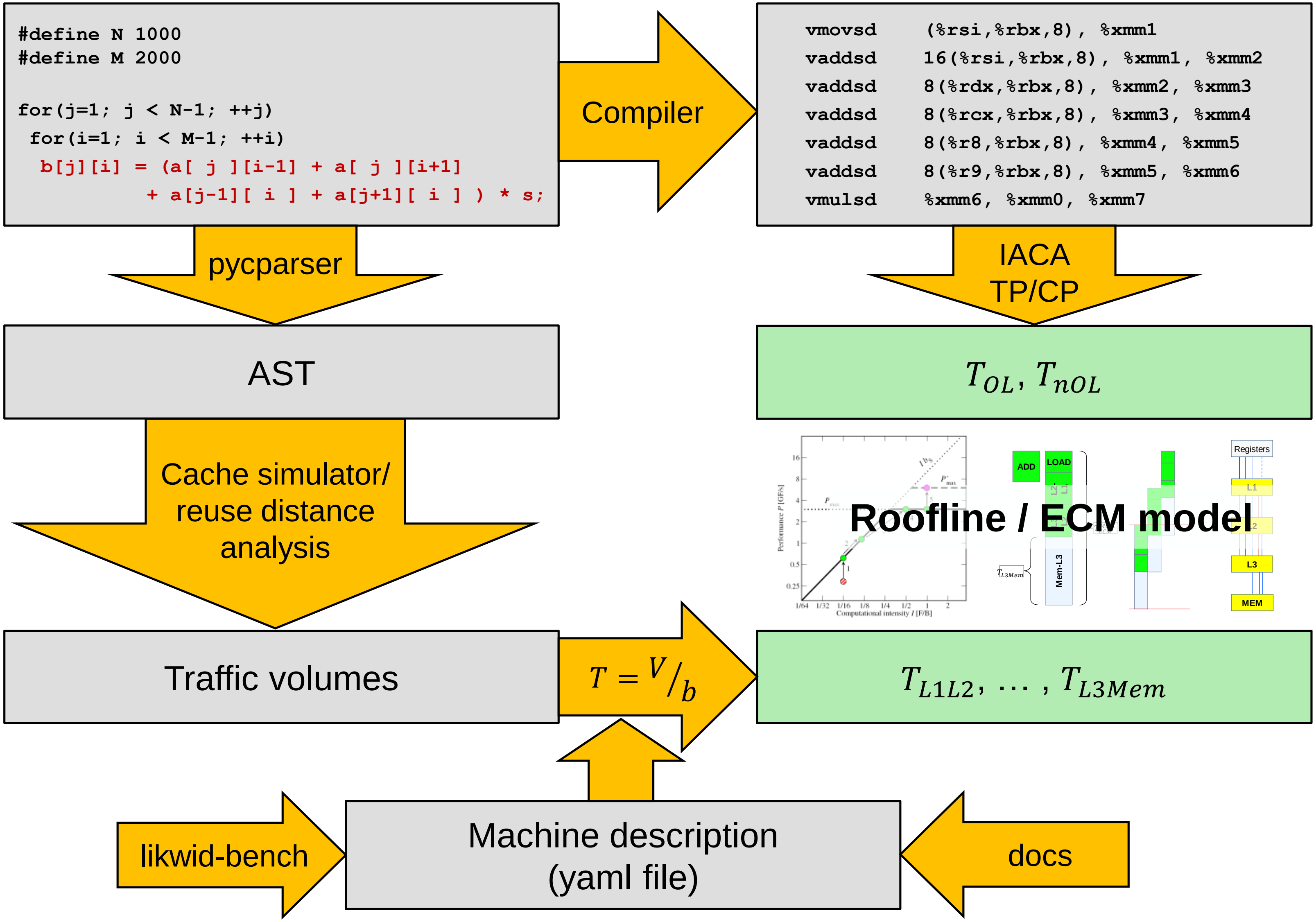
Additional requirements are:

- Intel IACA tool, with (working) `iaca.sh` in PATH environment variable (used by ECM, ECMCPU and Roofline models)
- likwid (used in Benchmark model and by `likwid_bench_auto.py`)

Towards automated model generation



kerncraft


#define N 1000
#define M 2000
for(j=1; j < N-1; ++j)
for(i=1; i < M-1; ++i)
b[j][i] = (a[j][i-1] + a[j][i+1]
+ a[j-1][i] + a[j+1][i]) * s;
Compiler
vmovsd (%rsi,%rbx,8), %xmm1
vaddsd 16(%rsi,%rbx,8), %xmm1, %xmm2
vaddsd 8(%rdx,%rbx,8), %xmm2, %xmm3
vaddsd 8(%rcx,%rbx,8), %xmm3, %xmm4
vaddsd 8(%r8,%rbx,8), %xmm4, %xmm5
vaddsd 8(%r9,%rbx,8), %xmm5, %xmm6
vmulsd %xmm6, %xmm0, %xmm7
pycparser
IACA TP/CP
AST
T_{OL}, T_{nOL}
Cache simulator/ reuse distance analysis
Roofline / ECM model
Registers
ADD
LOAD
L1
L3
MEM
Mem-L3
Traffic volumes
$T = {}^{V}/_{b}$
$T_{L1L2}, \ldots, T_{L3Mem}$
likwid-bench
Machine description (yaml file)
docs

Restrictions on code input (selection)

- Only doubles and ints supported
- Array declarations may use fixed sizes or constants, with an optional offset (e.g., double u1[M+3][N-2][23], but not double u[M*N])
- Only the innermost loop may contain assignment statements
- Array references must either use index variables from for-loops, with optional addition or subtraction, constant or fixed values
- All for-loops must use a declaration as initial statement and an increment or a decrement assignment operation as the next statement (e.g., i++, i -= 2)
- Function calls and the use of pointers is not allowed anywhere in the kernel code
- Write access to any data is assumed to use “normal” STORE instructions (e.g., no non-temporal stores)

Operating modes

- **`ECM`**
  - Full ECM model including in-core analysis
- **`ECMData`**
  - Data traffic analysis only (works on any system)
- **`ECMCPU`**
  - In-core part of ECM model (IACA)
- **`Roofline`**
  - Full Roofline model using CPU peak performance as in-core limit
- **`RooflineIACA`**
  - Full Roofline model using IACA analysis for in-core
- **`Benchmark`**
  - Run the actual benchmark for model validation

Machine file example: 8-core SNB EP node

```
clock: 2.7 GHz
cores per socket: 8
model type: Intel Core SandyBridge EP processor
model name: Intel(R) Xeon(R) CPU E5-2680 0 @ 2.70GHz
sockets: 2
threads per core: 2
cacheline size: 64 B
icc architecture flags: [-xAVX]
micro-architecture: SNB
FLOPs per cycle:
    SP: {total: 8, ADD: 4, MUL: 4}
    DP: {total: 4, ADD: 2, MUL: 2}
overlapping ports: ["0", "0DV", "1", "2", "3", "4", "5"]
non-overlapping ports: ["2D", "3D"]
memory hierarchy:
- {cores per group: 1, cycles per cacheline transfer: 2,
  groups: 16, level: L1, bandwidth: null, size per group: 32.00
    kB, threads per group: 2}
- {cores per group: 1, cycles per cacheline transfer: 2,
  groups: 16, level: L2, bandwidth: null, size per group: 256.00
    kB, threads per group: 2}
- {bandwidth per core: 18 GB/s, cores per group: 8, cycles per cacheline transfer: null,
  groups: 2, level: L3, bandwidth: 40 GB/s, size per group: 20.00
    MB, threads per group: 16}
- {cores per group: 8, cycles per cacheline transfer: null,
  level: MEM, bandwidth: null, size per group: null, threads per group: 16}
[…]
```

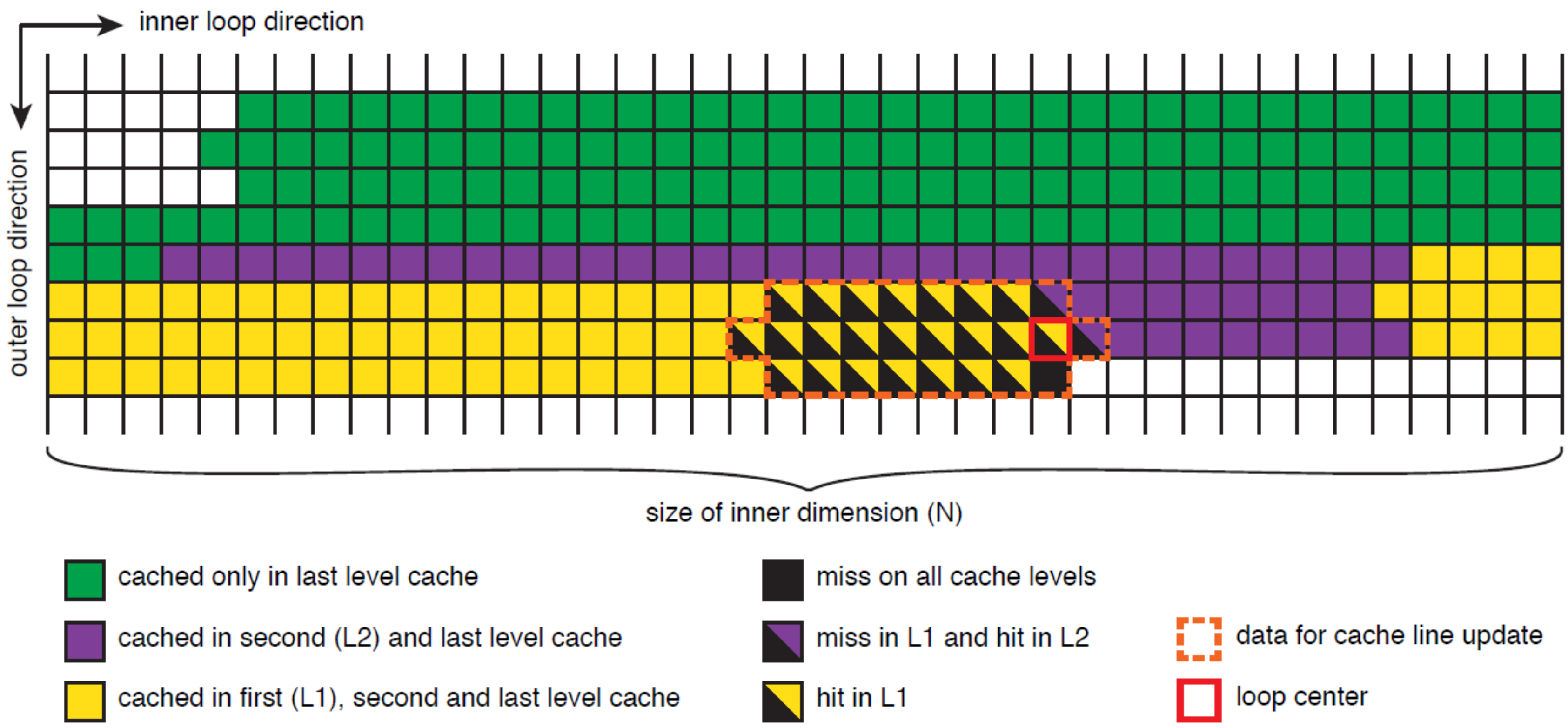
Machine file example (cont.)


```
benchmarks:
  kernels:
    copy:
      FLOPs per iteration: 0
      read streams: {bytes: 8.00 B, streams: 1}
      read+write streams: {bytes: 0.00 B, streams: 0}
      write streams: {bytes: 8.00 B, streams: 1}
    daxpy:
      FLOPs per iteration: 2
      read streams: {bytes: 16.00 B, streams: 2}
      read+write streams: {bytes: 8.00 B, streams: 1}
      write streams: {bytes: 8.00 B, streams: 1}
    load:
      FLOPs per iteration: 0
      read streams: {bytes: 8.00 B, streams: 1}
      read+write streams: {bytes: 0.00 B, streams: 0}
      write streams: {bytes: 0.00 B, streams: 0}
    triad:
      FLOPs per iteration: 2
      read streams: {bytes: 24.00 B, streams: 3}
      read+write streams: {bytes: 0.00 B, streams: 0}
      write streams: {bytes: 8.00 B, streams: 1}
    update:
      FLOPs per iteration: 0

[…]
```

Machine file example (cont.)


```
measurements:
[…]
MEM:
      1:
        cores: [1, 2, 3, 4, 5, 6, 7, 8]
        results:
          copy: [11.60 GB/s, 21.29 GB/s, 25.94 GB/s, 27.28 GB/s, 27.47 GB/s, 27.36
              GB/s, 27.21 GB/s, 27.12 GB/s]
          daxpy: [17.33 GB/s, 31.89 GB/s, 38.65 GB/s, 40.50 GB/s, 40.81 GB/s, 40.62
              GB/s, 40.59 GB/s, 40.26 GB/s]
          load: [12.01 GB/s, 23.04 GB/s, 32.79 GB/s, 40.21 GB/s, 43.39 GB/s, 44.14
              GB/s, 44.42 GB/s, 44.40 GB/s]
          triad: [12.73 GB/s, 24.27 GB/s, 30.43 GB/s, 31.46 GB/s, 31.77 GB/s, 31.74
              GB/s, 31.65 GB/s, 31.52 GB/s]
          update: [18.91 GB/s, 32.43 GB/s, 37.28 GB/s, 39.98 GB/s, 40.99 GB/s, 40.92
              GB/s, 40.61 GB/s, 40.34 GB/s]
        size per core: [40.00 MB, 20.00 MB, 13.33 MB, 10.00 MB, 8.00 MB, 6.67 MB,
          5.71 MB, 5.00 MB]
        size per thread: [40.00 MB, 20.00 MB, 13.33 MB, 10.00 MB, 8.00 MB, 6.67 MB,
          5.71 MB, 5.00 MB]
        threads: [1, 2, 3, 4, 5, 6, 7, 8]
        threads per core: 1
        total size: [40.00 MB, 40.00 MB, 40.00 MB, 40.00 MB, 40.00 MB, 40.00 MB, 40.00 MB, 40.00 MB,
40.00 MB, 40.00 MB]
```

Cache reuse analysis



kerncraft usage

```
$ kerncraft -h
usage: kerncraft [-h] [-v[v]]--machine MACHINE
                 --pmodel{ECM,ECMData,ECMCPU,Roofline,RooflineIACA,Benchmark}
                 [-D KEY VALUE] [--testcases] [--testcase-index INDEX]
                 [--verbose] [--asm-block BLOCK] [--store PICKLE]
                 [--ecm-plot ECM_PLOT]
                 FILE [FILE ...]
```

Examples:

```
$ kerncraft -vv -p ECM -m phinally.yaml 2d-5pt.c -D N 10000 -D M 10000
```

```
$ kerncraft -v -p Roofline -m phinally.yaml 2d-5pt.c -D N 10000 -D M 10000
```

kerncraft example (ECM)

```
$ kerncraft -vv -p ECM -m phinally.yaml 2d-5pt.c -D N 10000 -D M 10000
================================================================================
                                   2d-5pt.c
================================================================================
double a[M][N];
double b[M][N];
double s;

for(int j=1; j<M-1; ++j)
    for(int i=1; i<N-1; ++i)
        b[j][i] = ( a[j][i-1] + a[j][i+1]
                  + a[j-1][i] + a[j+1][i]) * s;

variables:     name |   type size
           ---------+-------------------------
                  a | double (10000, 10000)
                  s | double None
                  b | double (10000, 10000)
```

kerncraft example (ECM) continued

```
loop stack:               idx |       min        max       step
                    ---------+--------------------------------
                            j |         1       9999         +1
                            i |         1       9999         +1


data sources:            name |  offsets   ...
                    ---------+------------...
                            a | ('rel', 'j', 0), ('rel', 'i', -1)
                              | ('rel', 'j', 0), ('rel', 'i', 1)
                              | ('rel', 'j', -1), ('rel', 'i', 0)
                              | ('rel', 'j', 1), ('rel', 'i', 0)
                            s | ('dir',)


data destinations:       name |  offsets   ...
                    ---------+------------...
                            b | ('rel', 'j', 0), ('rel', 'i', 0)
```

kerncraft example (ECM) continued

```
FLOPs:       op | count
           ----+-------
             + |    3
             * |    1
                =======
                    4

constants:      name | value
           ---------+-----------
                   M | 10000
                   N | 10000

Ports and cycles: {'1': 6.0, '0DV': 0.0, '2D': 8.0, '0': 5.05, '3': 9.0, '2':
9.0, '5': 5.95, '4': 4.0, '3D': 8.0}
Uops: 37.0
Throughput: 9.45cy per CL
T_nOL = 8.0cy
T_OL = 9.0cy
```

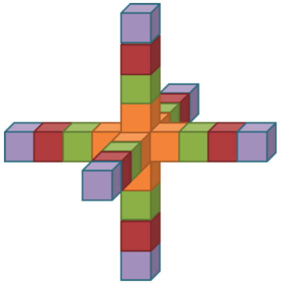
kerncraft example (ECM) continued

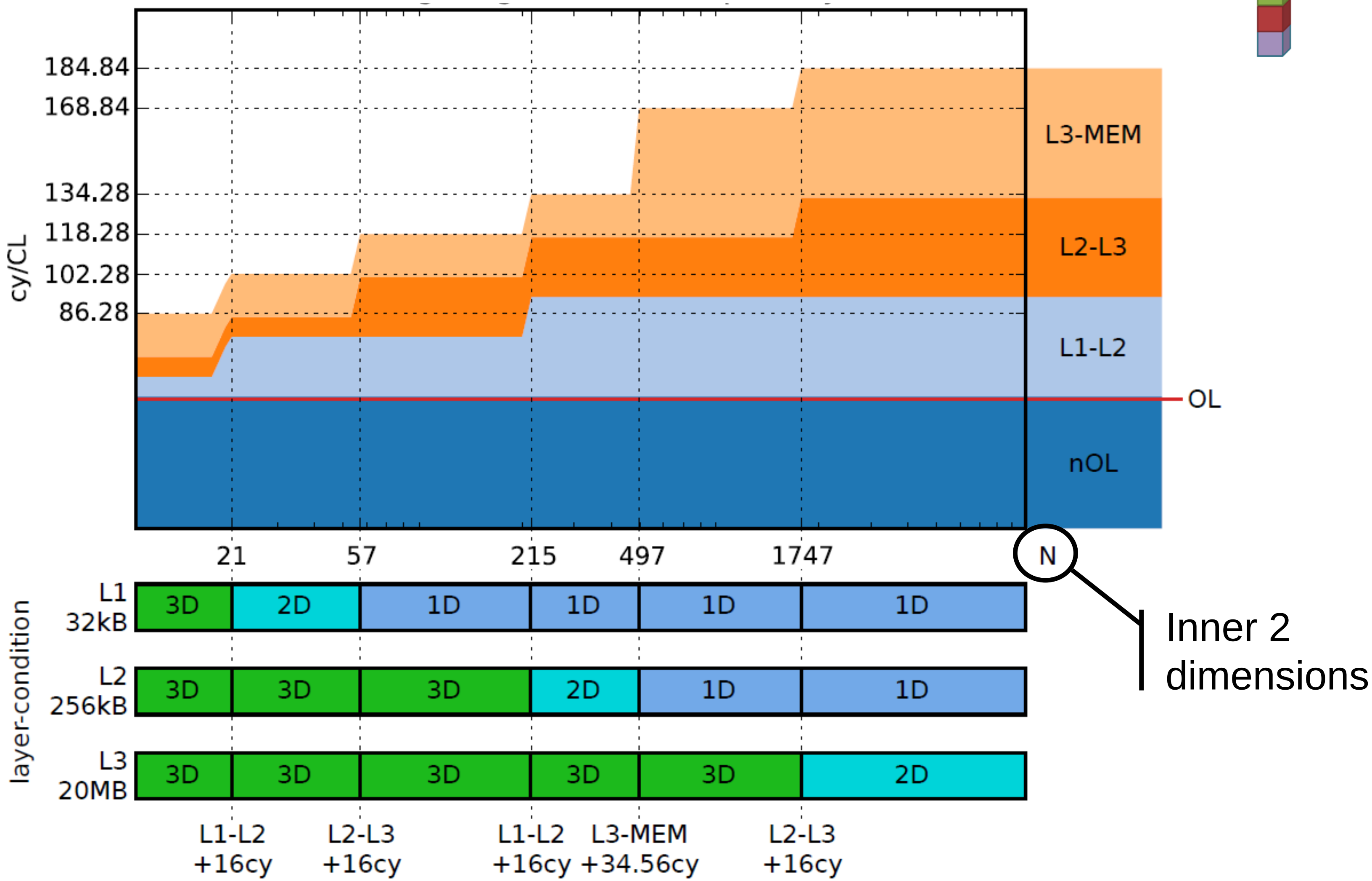
```
Trace length per access in L1: 982
Hits in L1: 30 {'a': {'ji': [10006, 10005, 10004, 10003, 10002, 10001, 10000,
7, 6, 5, 4, 3, 2, 1, 0, -1, -9994, -9995, -9996, -9997, -9998, -9999, -10000]},
's': {}, 'b': {'ji': [6, 5, 4, 3, 2, 1, 0]}}
Misses in L1: 4 (4CL): {'a': {'ji': [10007, 8, -9993]}, 's': {}, 'b': {'ji':
[7]}}
Evicts from L1 8 (1CL): {'a': {}, 's': {}, 'b': {'ji': [7, 6, 5, 4, 3, 2, 1,
0]}}

...

L1-L2 = 10cy
L2-L3 = 10cy
L3-MEM = 12.96cy
{ 9.0 || 8.0 | 10 | 10 | 12.96 } cy
{ 9.0 \ 18 \ 28 \ 41 } cy
```

kerncraft example (Roofline)

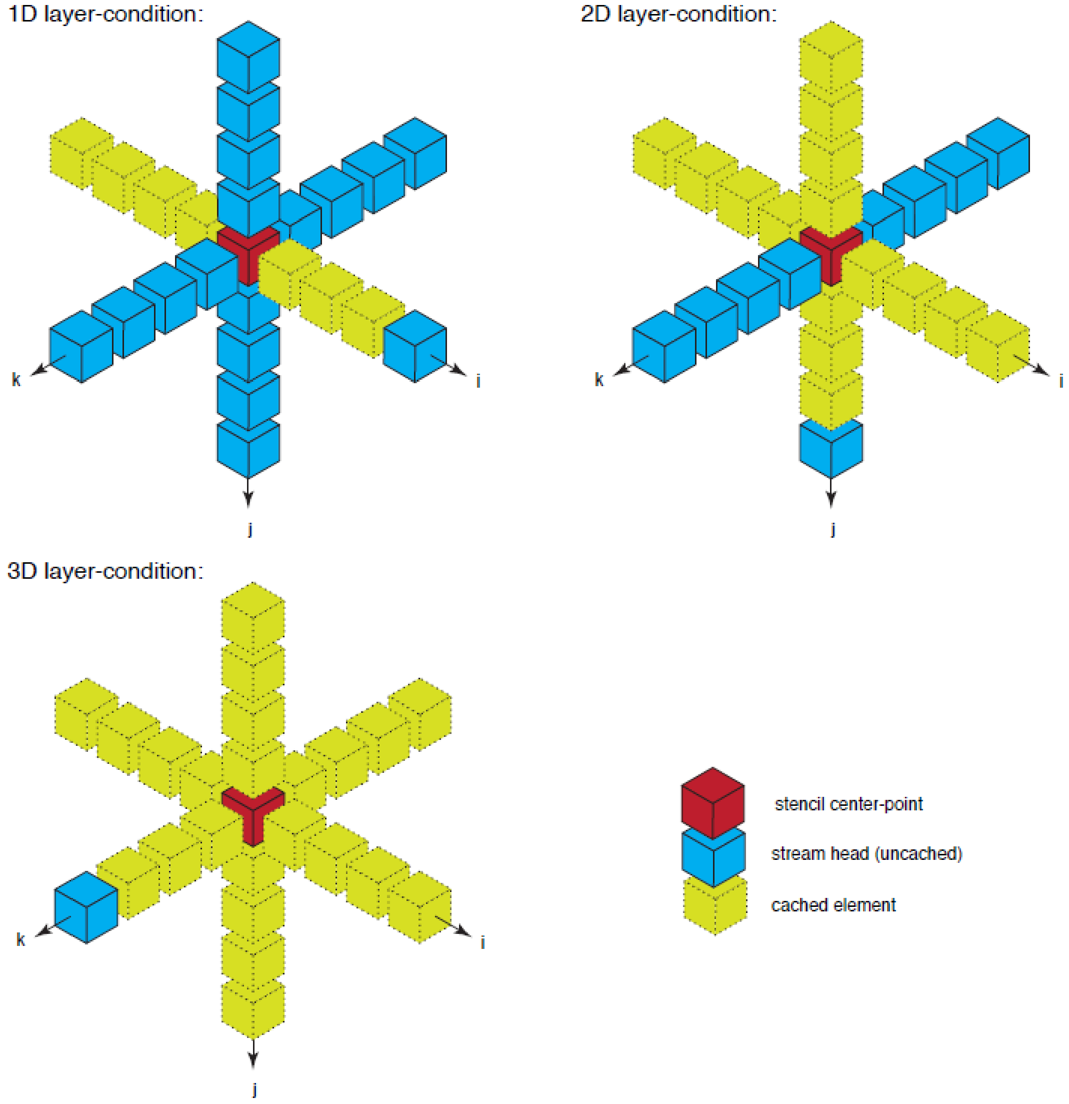
```
$ kerncraft -v -p Roofline -m phinally.yaml 2d-5pt.c -D N 10000 -D M 10000

...

Bottlenecks:

  level | a. intensity |   performance    |   bandwidth   | bandwidth kernel
--------+--------------+-----------------+---------------+-----------------
    CPU |              |   21.60 GFLOP/s |               |
 CPU-L1 | 0.083 FLOP/b |    8.50 GFLOP/s |  102.01 GB/s  | triad
  L1-L2 |   0.1 FLOP/b |    5.12 GFLOP/s |   51.15 GB/s  | triad
  L2-L3 |   0.1 FLOP/b |    3.15 GFLOP/s |   31.48 GB/s  | triad
 L3-MEM |  0.17 FLOP/b |    2.90 GFLOP/s |   17.40 GB/s  | copy

Cache or mem bound
2.90 GFLOP/s due to L3-MEM transfer bottleneck (bw with from copy benchmark)
Arithmetic Intensity: 0.17 FLOP/b
```

Interpretation of predictions: 3D long-range stencil

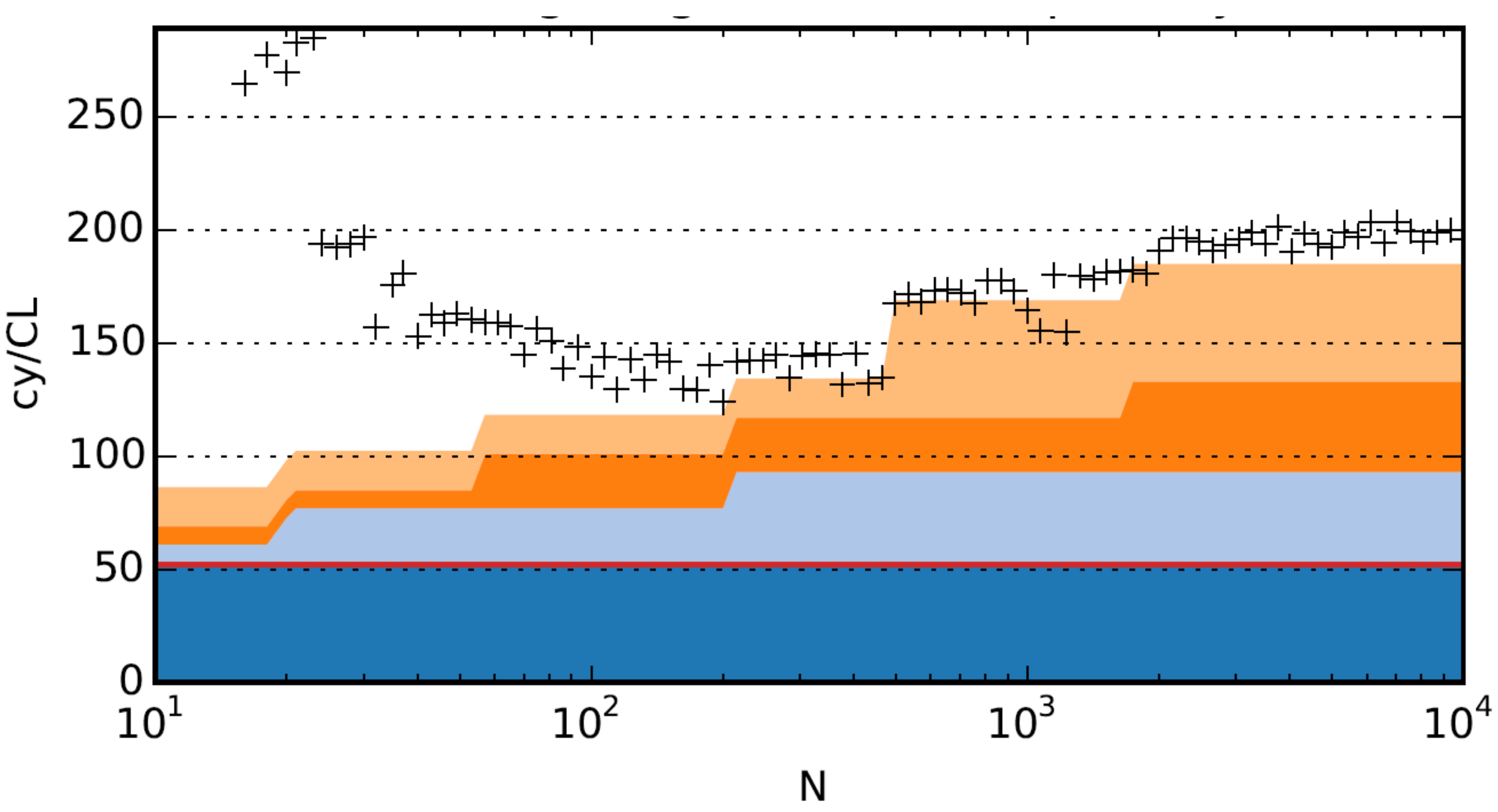




Layer conditions in the 3D long-range stencil



Comparison of measurements with predictions: 3D long-range stencil



Summary & remarks

- No silver bullet
  - Tool output must be checked
  - Validation is absolutely mandatory
  - If the model does not work, we learn something

- Future work
  - Lift some of the restrictions on the C formulation of the loop code
  - Include saturation analysis
  - Become more independent of external tools
    - IACA, icc
  - Improve simplistic reuse analysis

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Thank You.

Julian Hammer
Johannes Hofmann
Holger Stengel
Jan Eitzinger