Compiler Generation and Autotuning of Communication-Avoiding Operators for Geometric Multigrid

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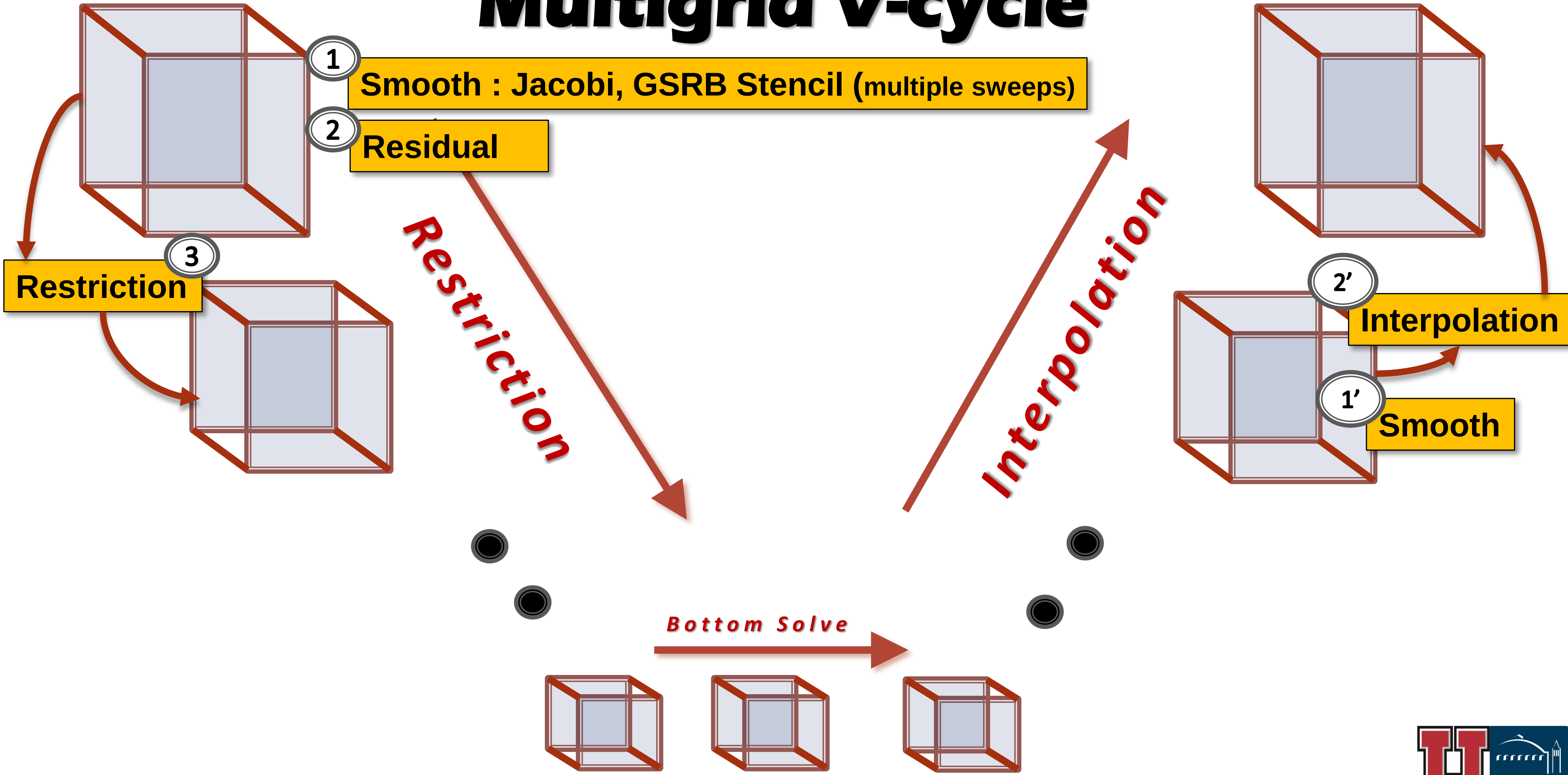
http://ctop.cs.utah.edu/x-tune/

Multigrid V-cycle



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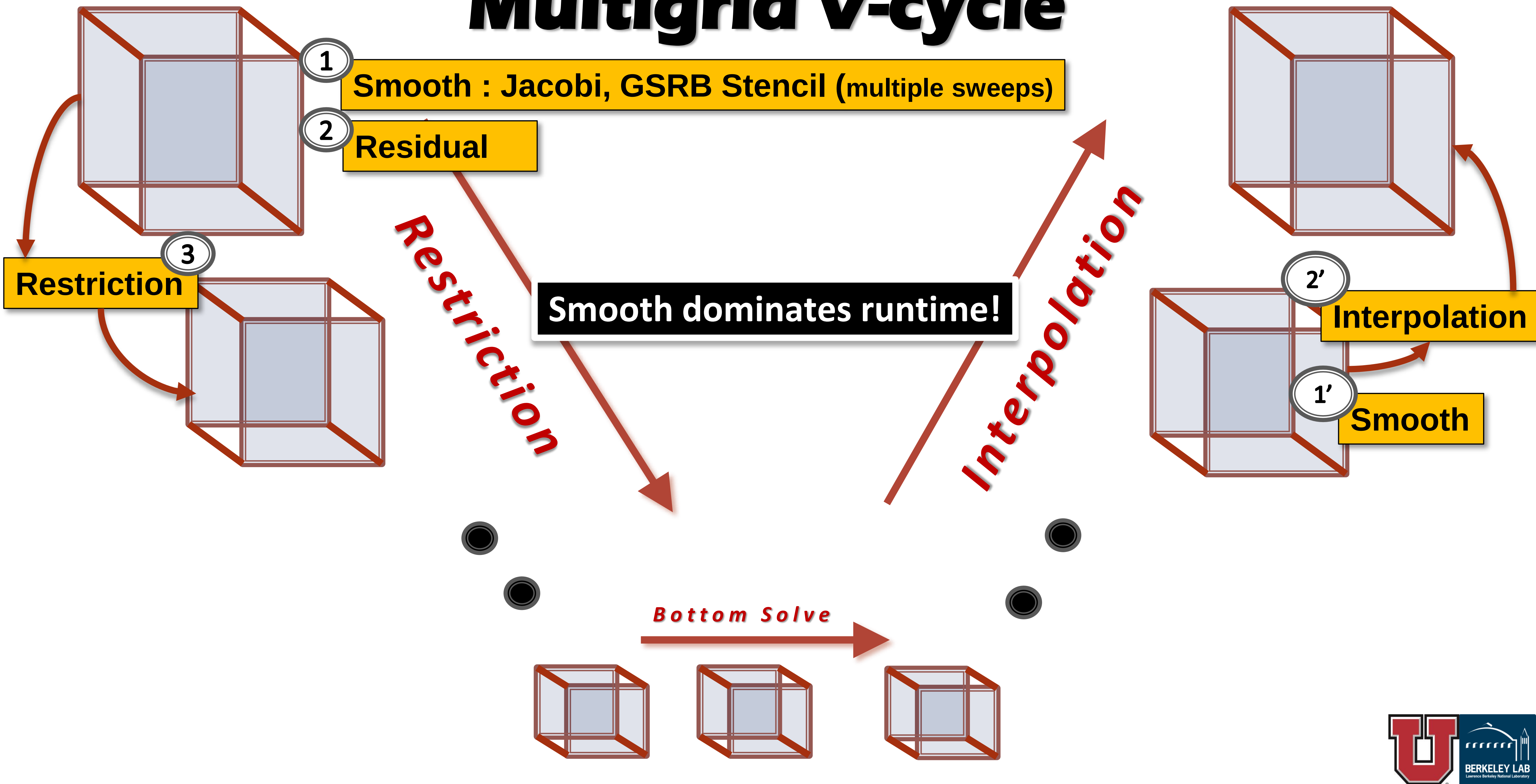


Compiler-Based Optimization of Smooth

Optimization Using Known Transformations

- Loop skew, permute and tiling

New Domain Specific Transformations

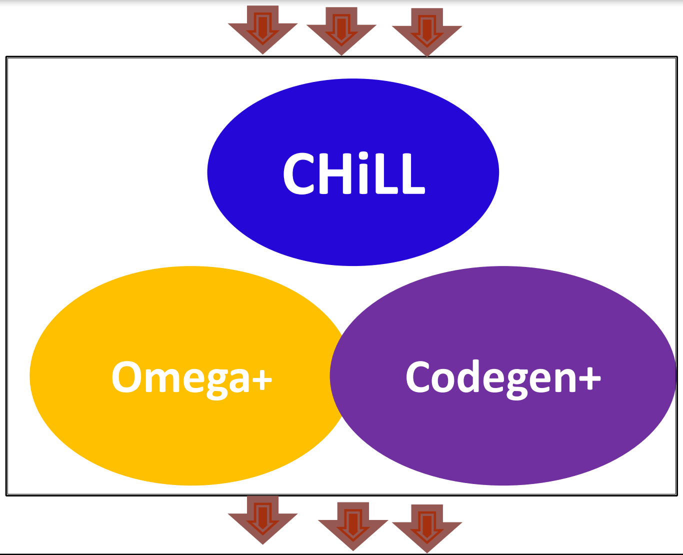
- Loop fusion in presence of fusion preventing dependences
- Adding ghost zones (communication avoiding) to Multigrid operators

High Performance OpenMP Code Generation

Optimizations Built into CHiLL

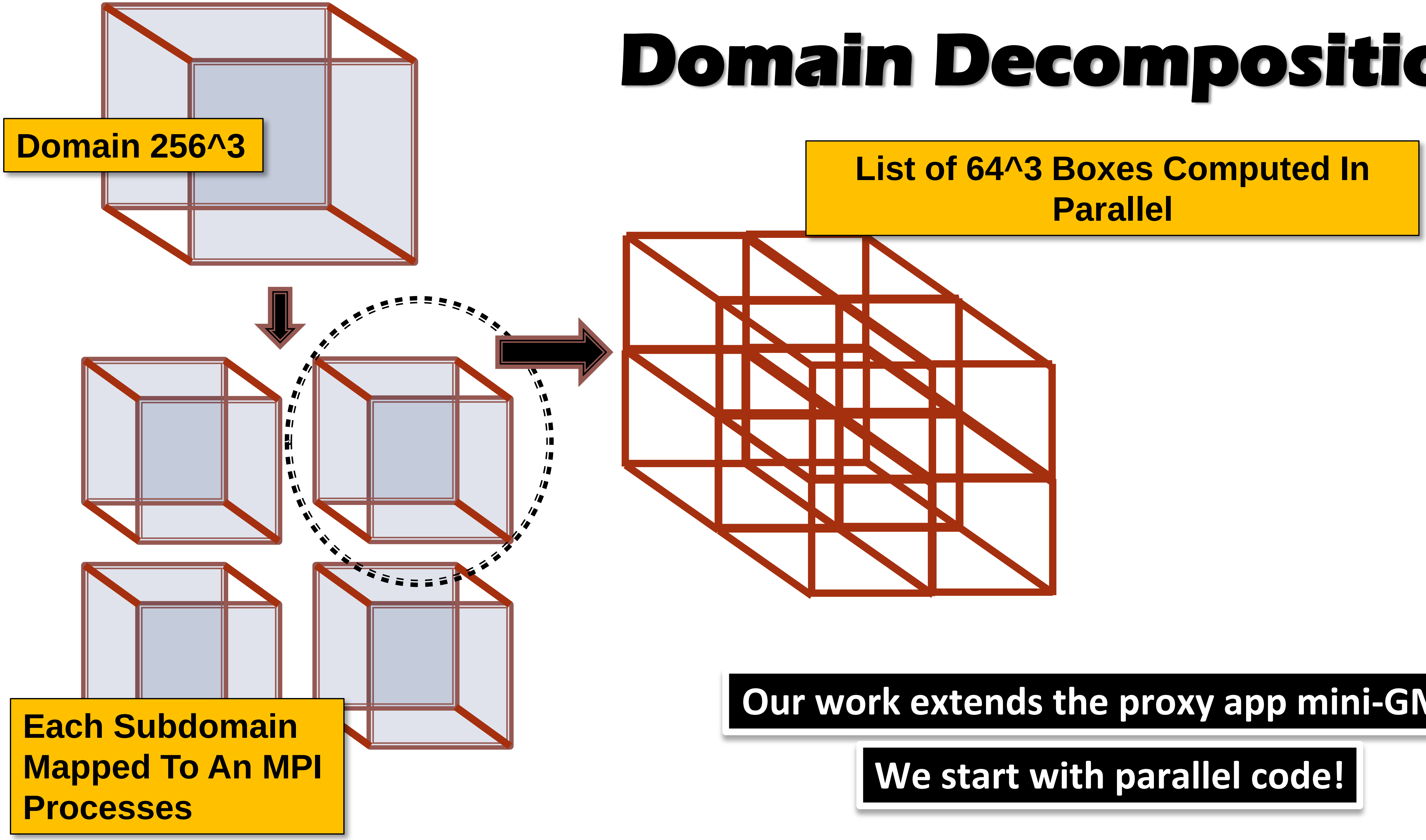
- CHiLL is loop transformation framework with a script interface

Autotuner



Smooth Variants

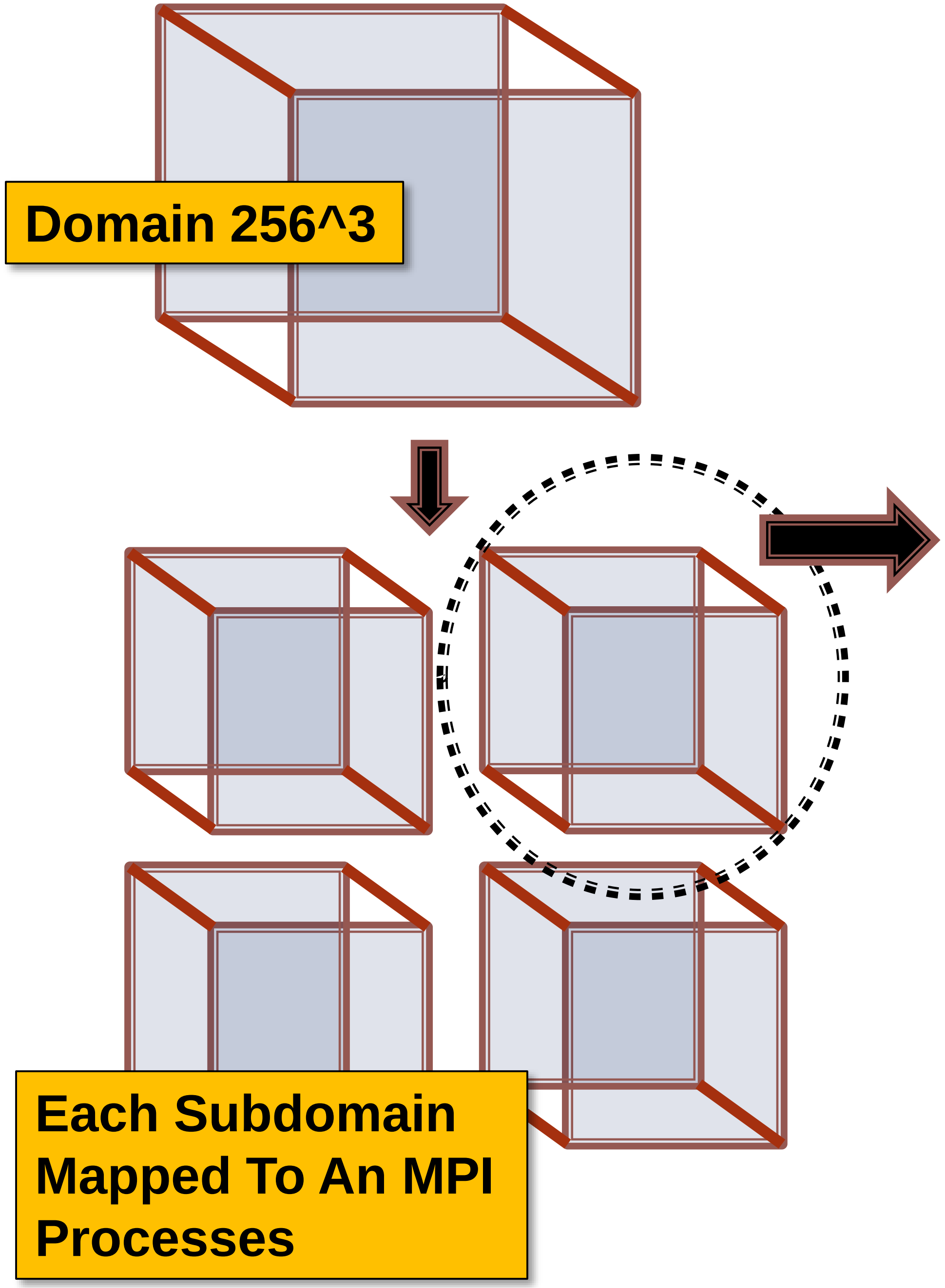
Domain Decomposition



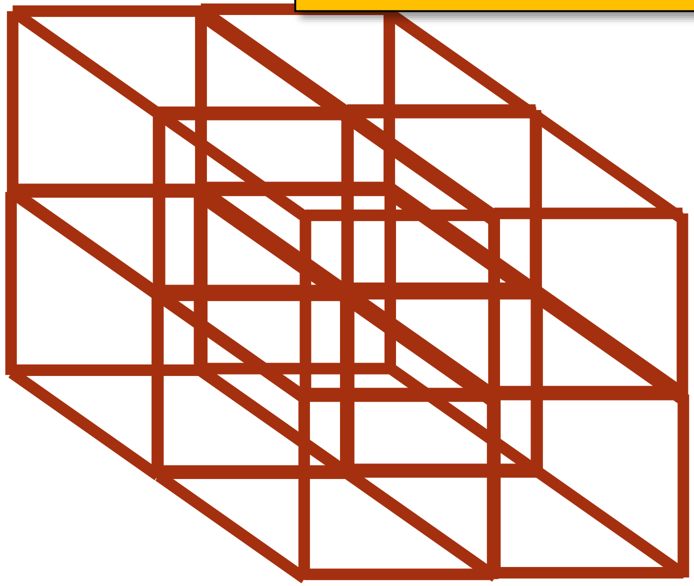
Our work extends the proxy app mini-GMG

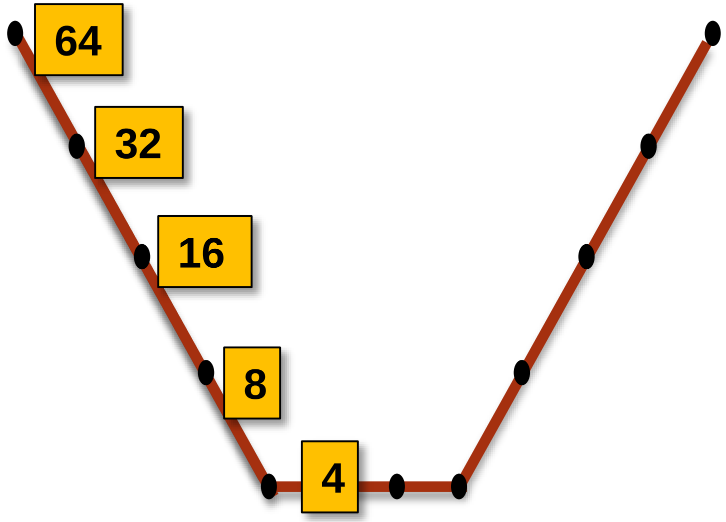
We start with parallel code!

Domain Decomposition



List of 64^3 Boxes Computed In Parallel





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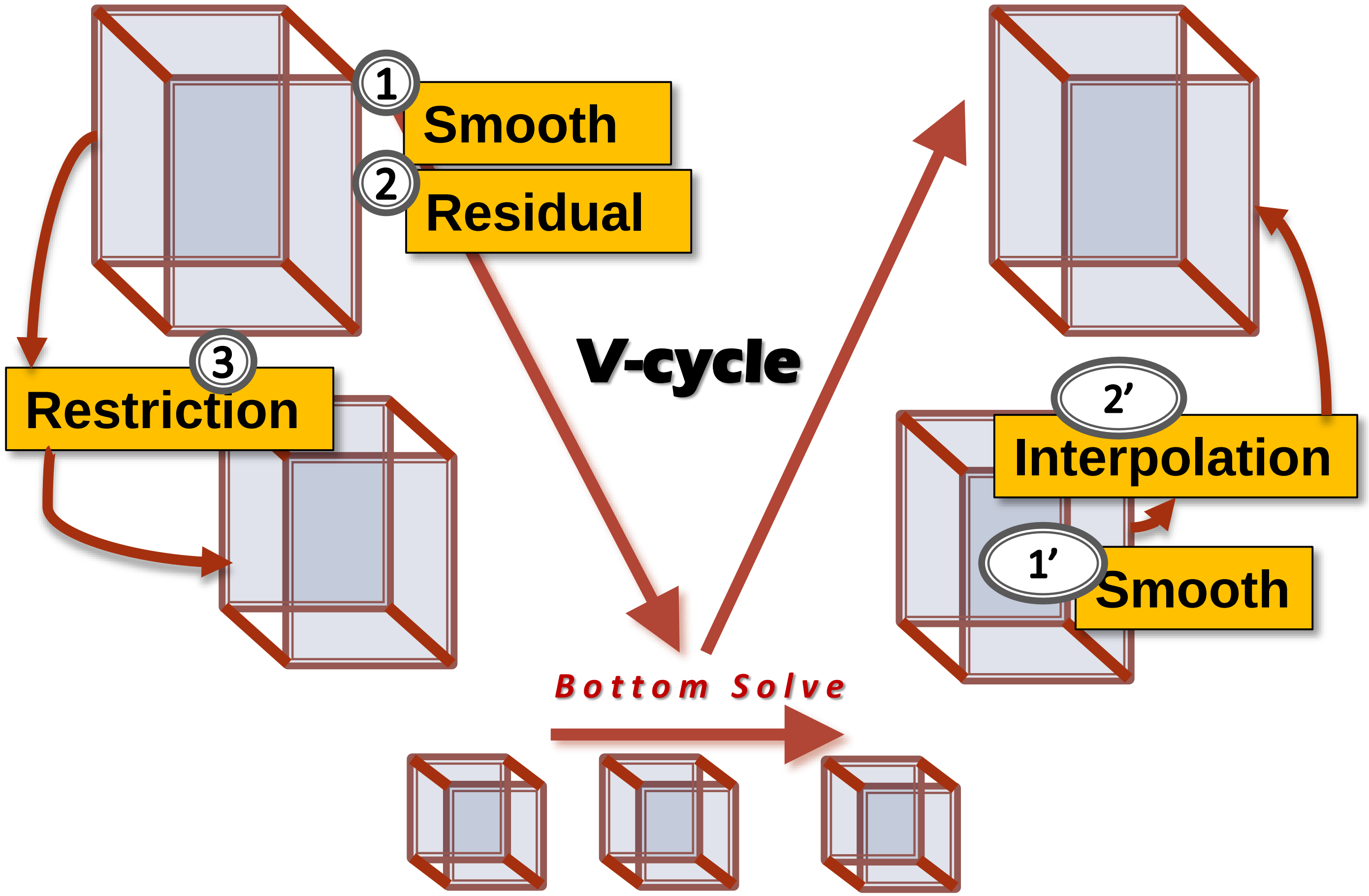
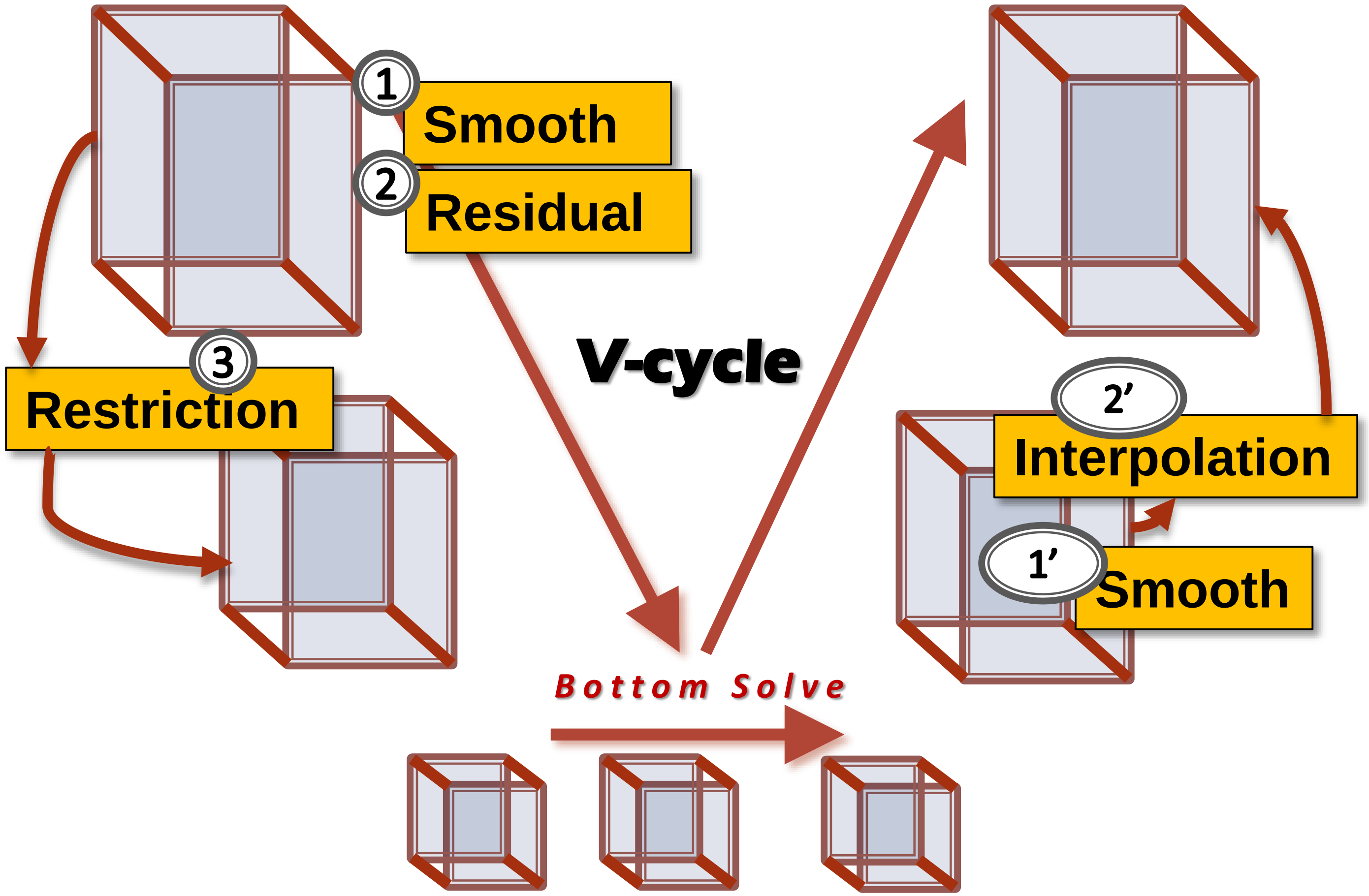


Solve : Lu = f

Double precision, finite volume discretization of the variable-coefficient operator
$$L = a\vec{\alpha}I - b\nabla\vec{\beta}\nabla$$
with periodic boundary conditions

Right hand side (f) is:
$$\sin(2\pi x)\sin(2\pi y)\sin(2\pi z)$$
on the [0,1] cubical domain



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sin(2πx)sin(2πy)sin(2πz)
on the [0,1] cubical domain

8 GSRB sweeps per level of the V-cycle (4 going down and 4 coming back up)

48 GSRB sweeps at the bottom level

Smooth Operator

```
/* Laplacian(phi) = b div beta grad phi */
for (k=0; k<N; k++)
   for (j=0; j<N; j++)
      for (i=0; i<N; i++)
       /* statement S0 */
       temp[k][j][i] = b * h2inv * (
       beta_i[k][j][i+1] * ( phi[k][j][i+1] – phi[k][j][i] )
       -beta_i[k][j][i] * ( phi[k][j][i] – phi[k][j][i-1] )
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```

```
/* Helmholtz(phi) = (a alpha I – Laplacian) * phi */
for (k=0; k<N; k++)
   for (j=0; j<N; j++)
      for (i=0; i<N; i++)
       /* statement S1 */
       temp[k][j][i] = a * alpha[k][j][i] * phi[k][j][i] – temp[k][j][i];
```

```
/* GSRB relaxation  phi = phi – lambda(Helmholtz – rhs) */
for (k=0; k<N; k++)
   for (j=0; j<N; j++)
      for (i=0; i<N; i++){
       if ((i+j+k+color)%2 == 0 )
       /* statement S2 */
       phi[k][j][i] = phi[k][j][i] – lambda[k][j][i] *(temp[k][j][i] – rhs[k][j][i]);}
```

```
for (k...){ for (j...){ for (i...) { // LAPLACIAN OPERATOR }}}
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Smooth Operator

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```

3D 7-Point, Variable Coefficient Stencil

```
/* Laplacian(phi) = b div beta grad phi */
for (k=0; k<N; k++)
  for (j=0; j<N; j++)
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3D 7-Point, Variable Coefficient Stencil

Smooth Operator Optimization

Loop fusion across operators
Introduce ghost zones (halo regions)
Create a wavefront computation
OpenMP parallel code generation

Loop Fusion

```
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Loop Fusion

7-point stencil

Fusion-preventing dependences

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7-point stencil

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Solution: Array Data-Flow

Array data-flow analysis (temp) determines it is safe to contract iteration space for Laplacian and Helmholtz.

Loop Fusion

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7-point stencil

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Fused

```
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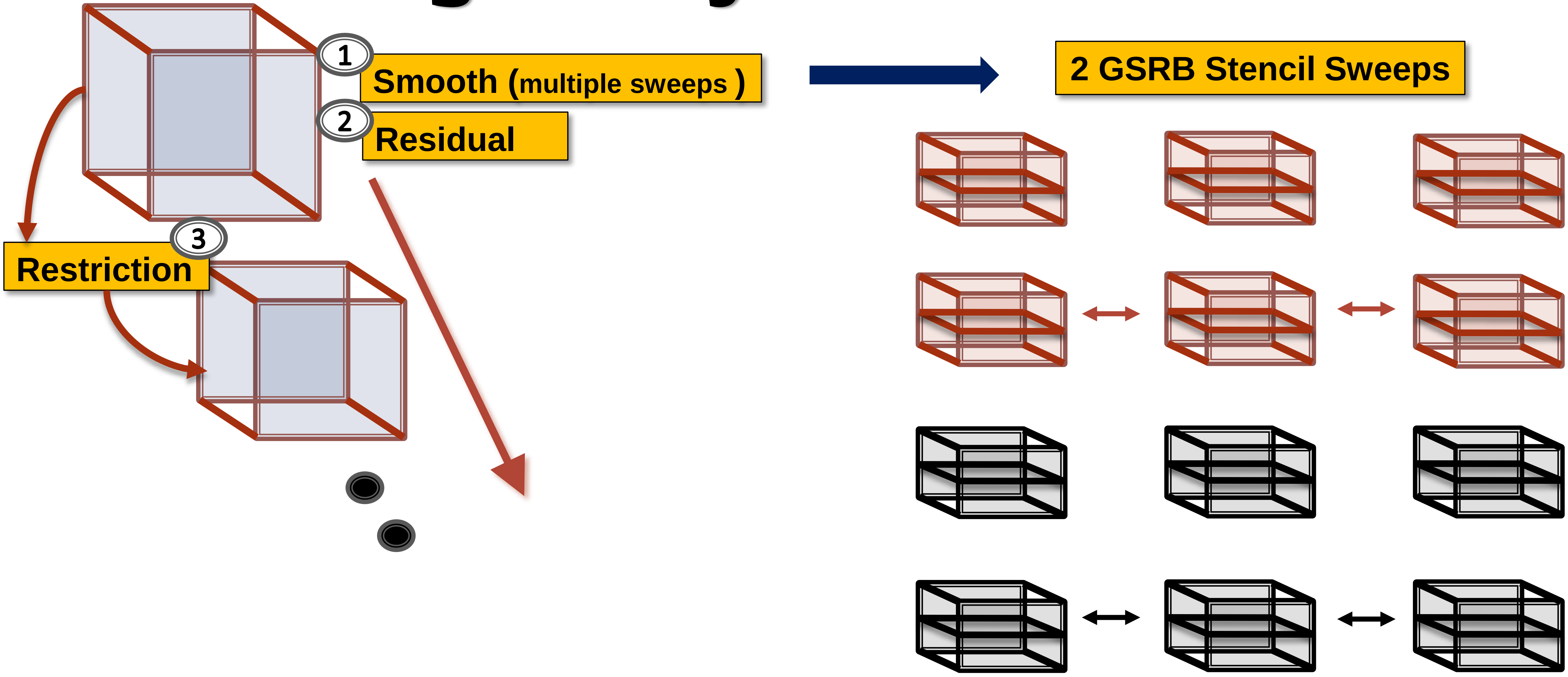
   for (j=0; j<N; j++)
      for (i=0; i<N; i++) {
        if ((i+j+k+color)%2 == 0) {
          S0(k,j,i); /* Laplacian  */
          S1(k,j,i); /* Helmholtz */
          S2(k,j,i); /* GSRB       */
      }
```

Multigrid V-cycle : Closer Look



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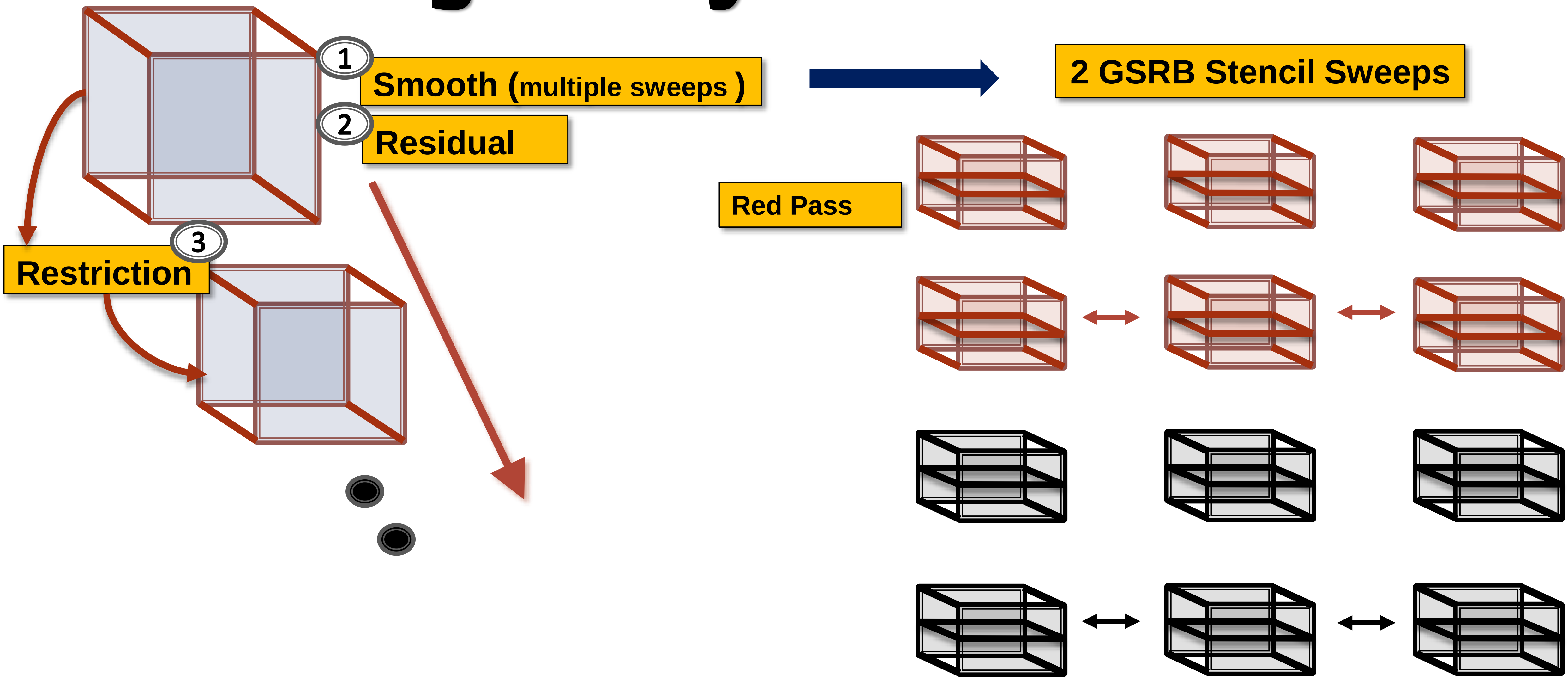

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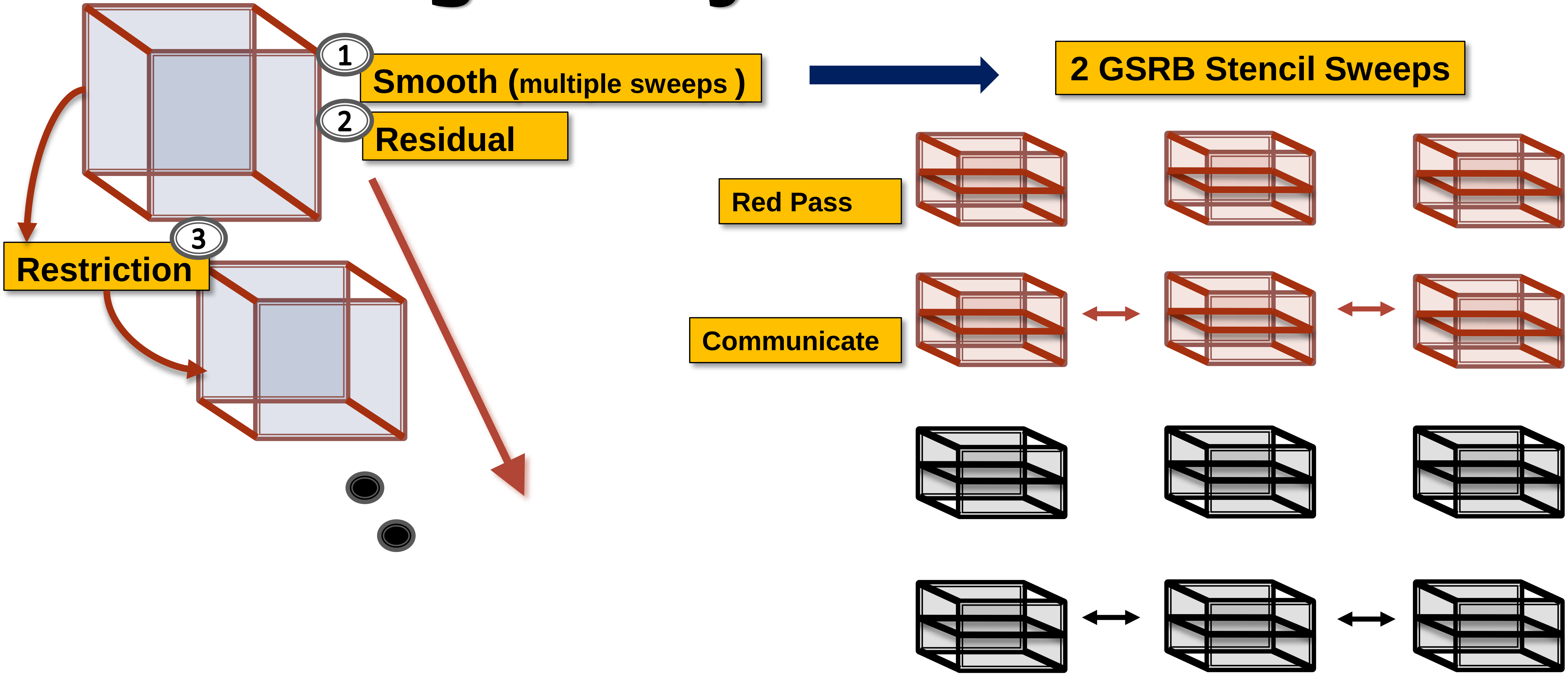

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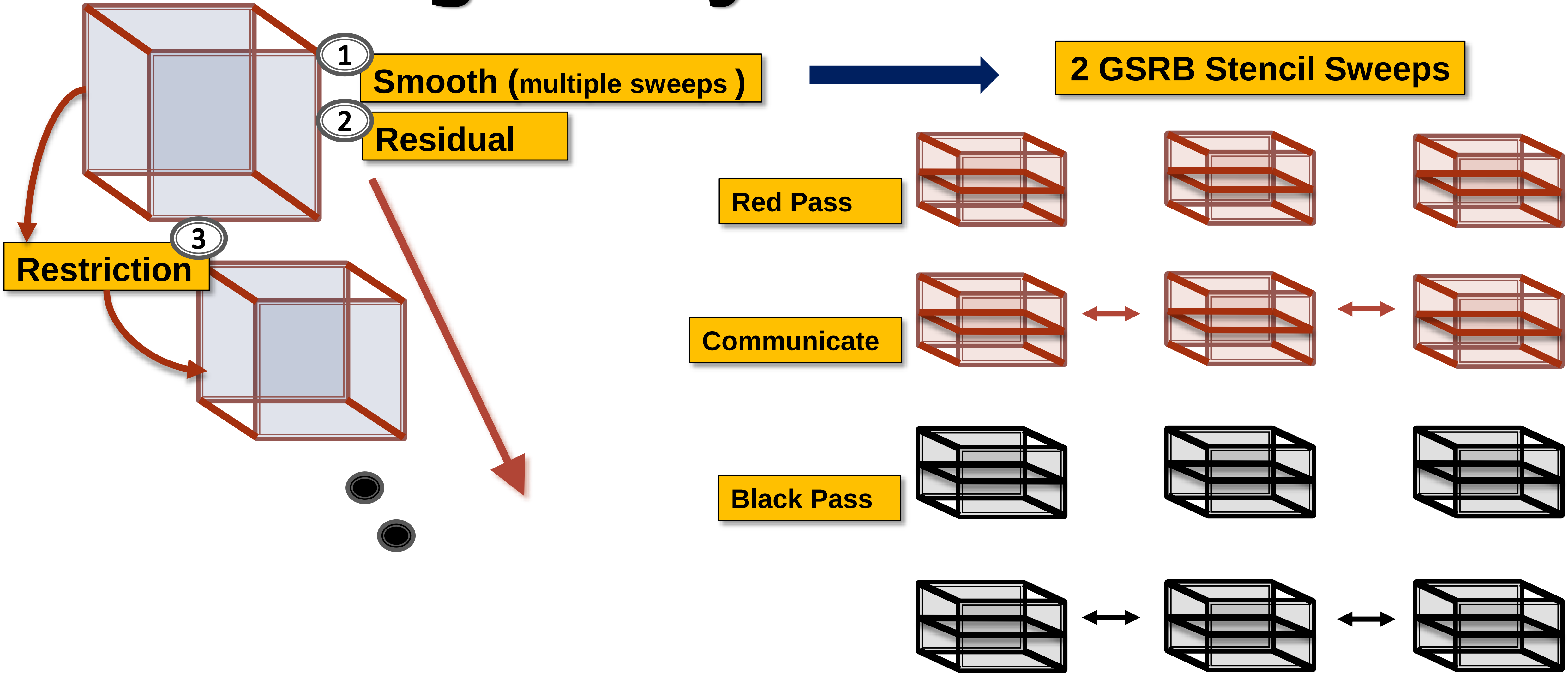

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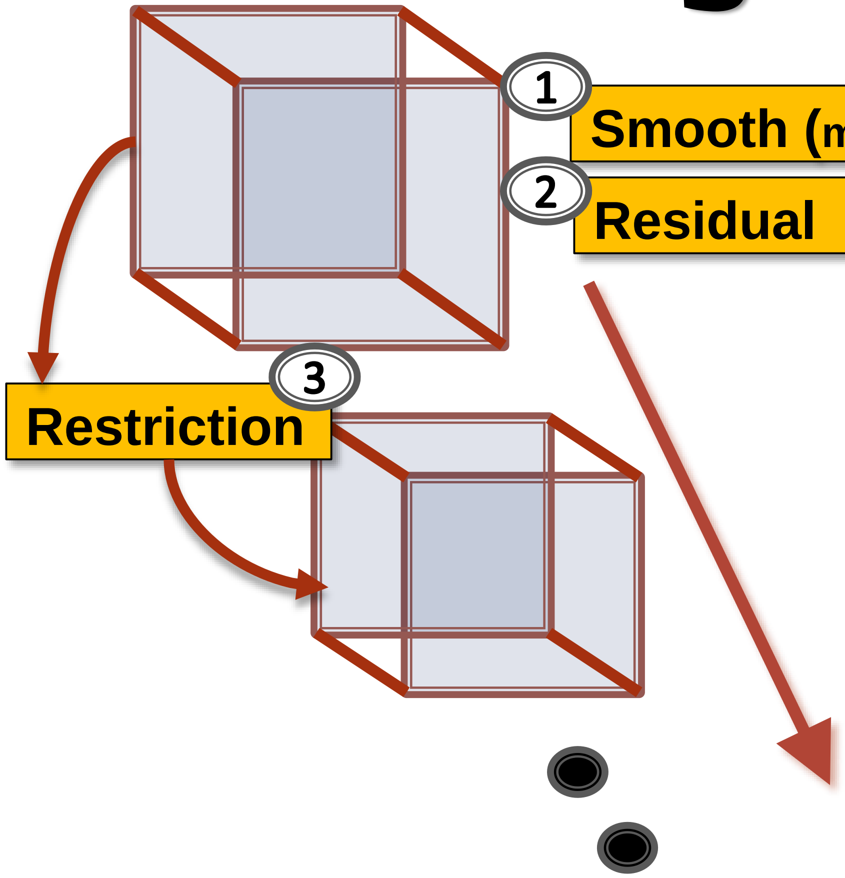
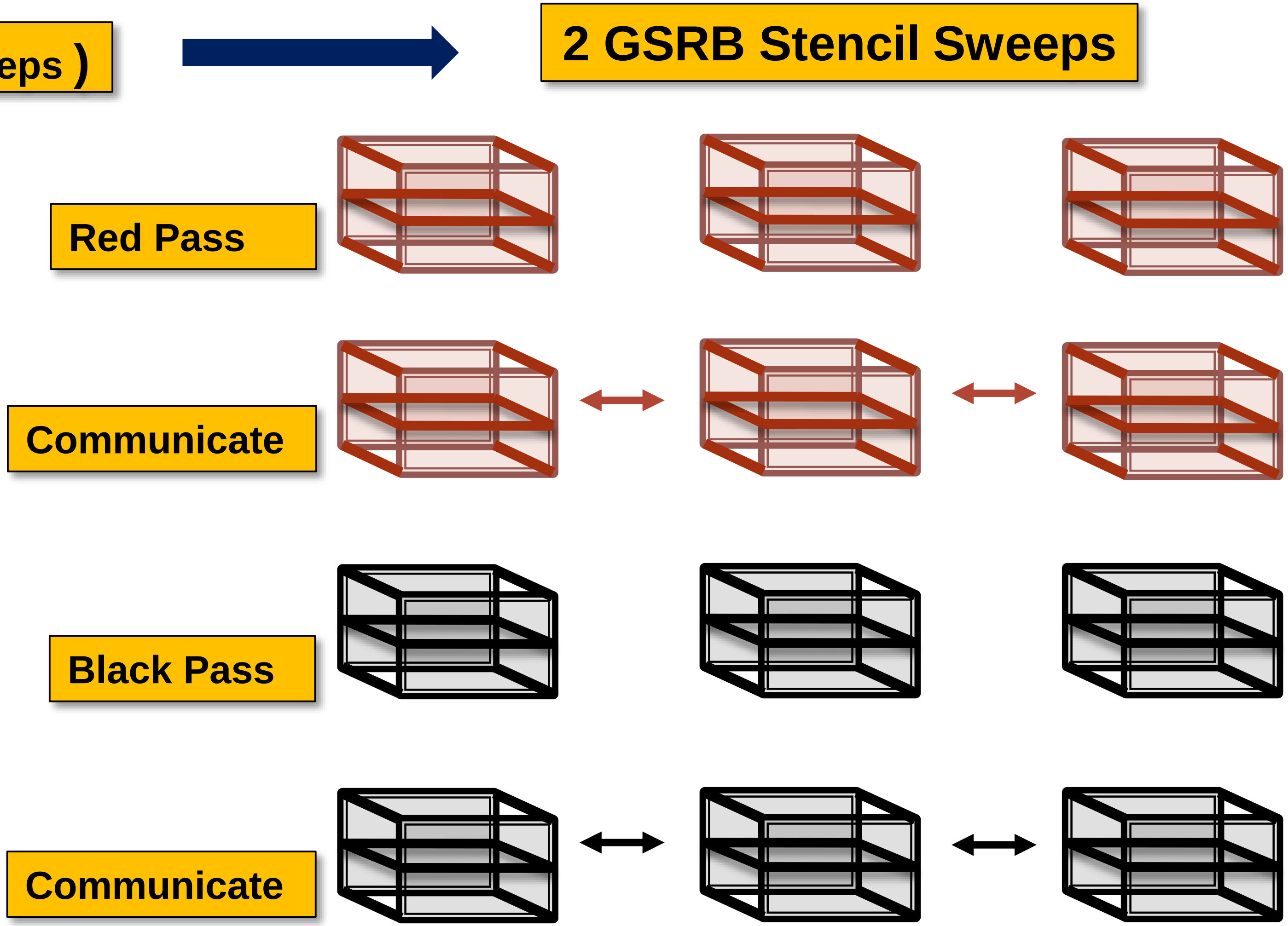
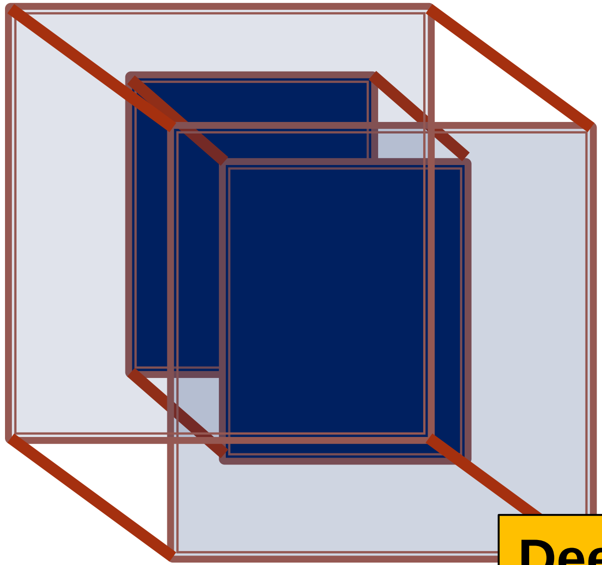




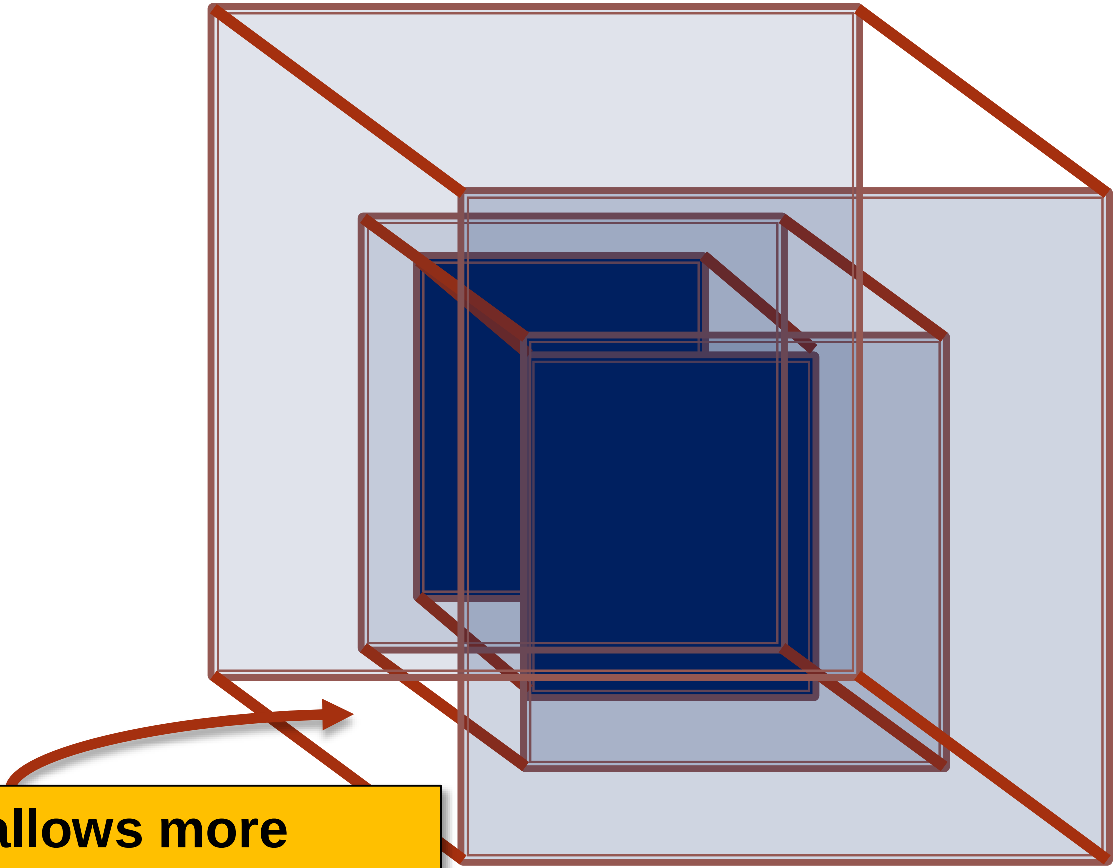
Adding Ghost Zones : Compute More, Communicate Less

Each grid sweep consumes one ghost layer

One communication step per sweep



Deeper ghost zone allows more sweeps per communication step



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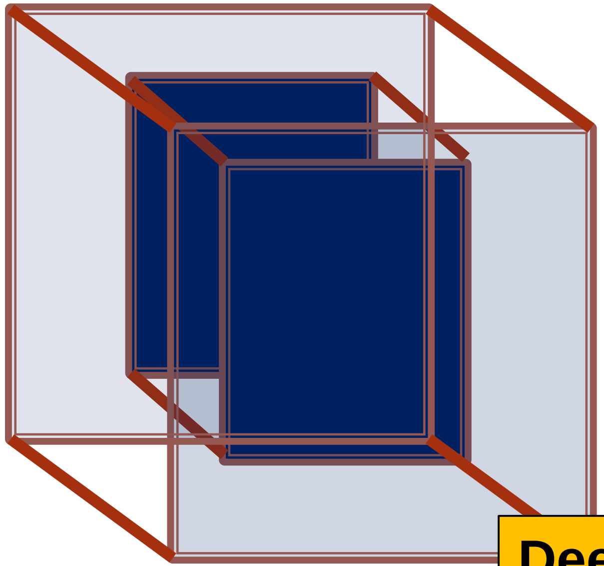

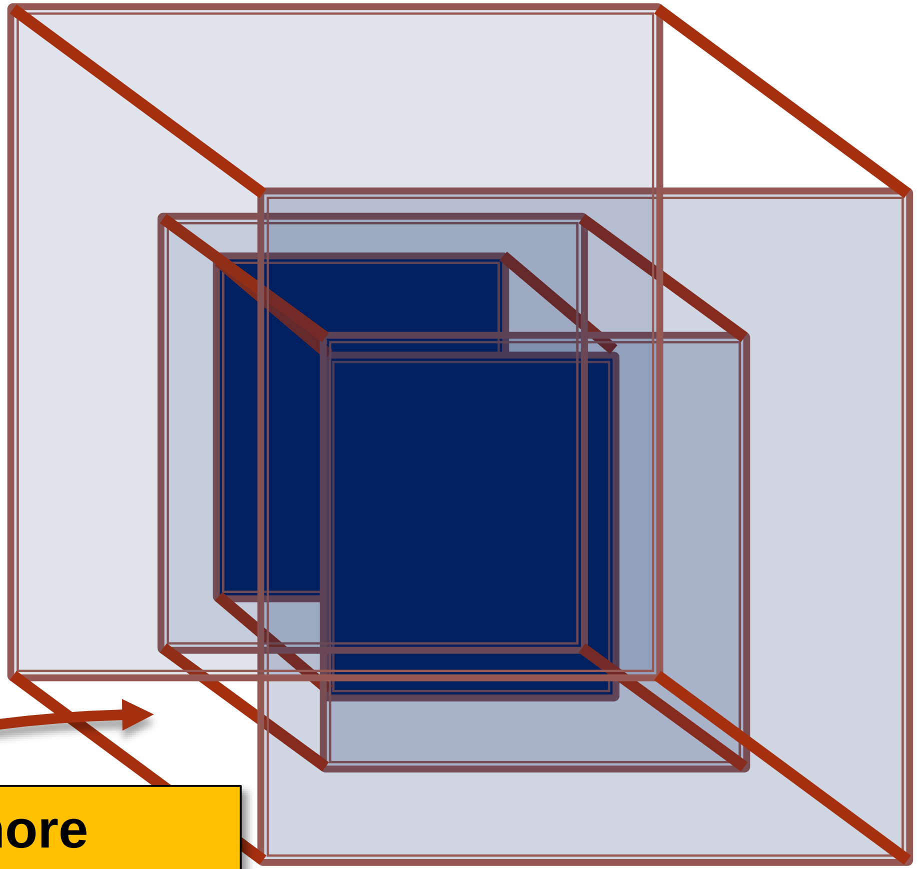

Deeper ghost zone allows more sweeps per communication step

Ghost zone depth depends on box size and machine!

Adding Ghost Zone

Fused

```
for (k=0; k<N; k++)
   for (j=0; j<N; j++)
      for (i=0; i<N; i++) {
         if ((i+j+k+color)%2 == 0) {
            S0(k,j,i); /* Laplacian  */
            S1(k,j,i); /* Helmholtz */
            S2(k,j,i); /* GSRB       */
         }
```

Ghost Zones

```
/* d = ghost zone depth */
for (t=0; t<d; t++)
   for (k=t-(d-1); k<N+(d-1)-t; k++)
      for (j=t-(d-1); j<N+(d-1)-t; j++)
         for (i=t-(d-1); i<N+(d-1)-t; i++) {
            if ((i+j+k+color+t)%2 == 0) {
               S0(k,j,i); /* Laplacian  */
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            }
```

Add Time Step Loop

Vary Iteration Space with Time

Update Conditional

Adding Ghost Zone

Fused

```
for (k=0; k<N; k++)
   for (j=0; j<N; j++)
      for (i=0; i<N; i++) {
         if ((i+j+k+color)%2 == 0) {
            S0(k,j,i); /* Laplacian  */
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         }
```

Solution

Expand iteration space (compiler abstraction) to include ghost zone. Code generation involves scanning polyhedra described by iteration space

Ghost Zones

```
/* d = ghost zone depth */
for (t=0; t<d; t++)
   for (k=t-(d-1); k<N+(d-1)-t; k++)
      for (j=t-(d-1); j<N+(d-1)-t; j++)
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```

Add Time Step Loop

Vary Iteration Space with Time

Update Conditional

Adding Ghost Zone

Fused

```
for (k=0; k<N; k++)
   for (j=0; j<N; j++)
      for (i=0; i<N; i++) {
         if ((i+j+k+color)%
            S0(k,j,i); /* Laplacian  */
            S1(k,j,i); /* Helmholtz */
            S2(k,j,i); /* GSRB       */
         }
```

Original Iteration Space

S0 := { [k,j,i] : 0 ≤ k < N && 0 ≤ j < N && 0 ≤ i < N && k+j+I + 2α + color = 0};

Relations to Map to New Iteration Space

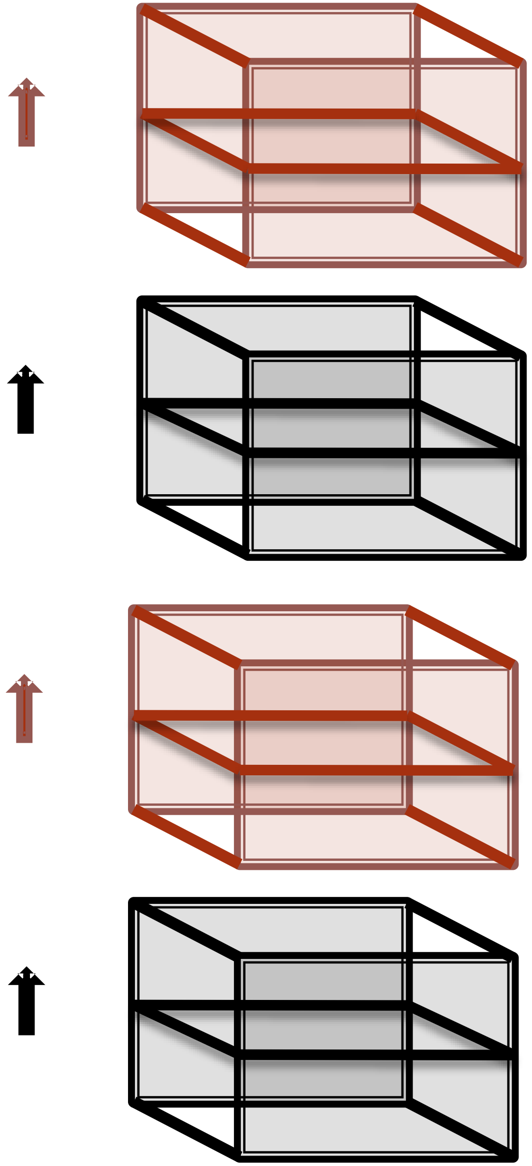
Ghost Zones

```
/* d = ghost zone depth */
for (t=0; t<d; t++)
   for (k=t-(d-1); k<N+(d-1)-t; k++)
      for (j=t-(d-1); j<N+(d-1)-t; j++)
         for (i=t-(d-1); i<N+(d-1)-t; i++) {
            if ((i+j+k+color+t)%2 == 0) {
               S0(k,j,i); /* Laplacian  */
               S1(k,j,i); /* Helmholtz */
               S2(k,j,i); /* GSRB       */
            }
```

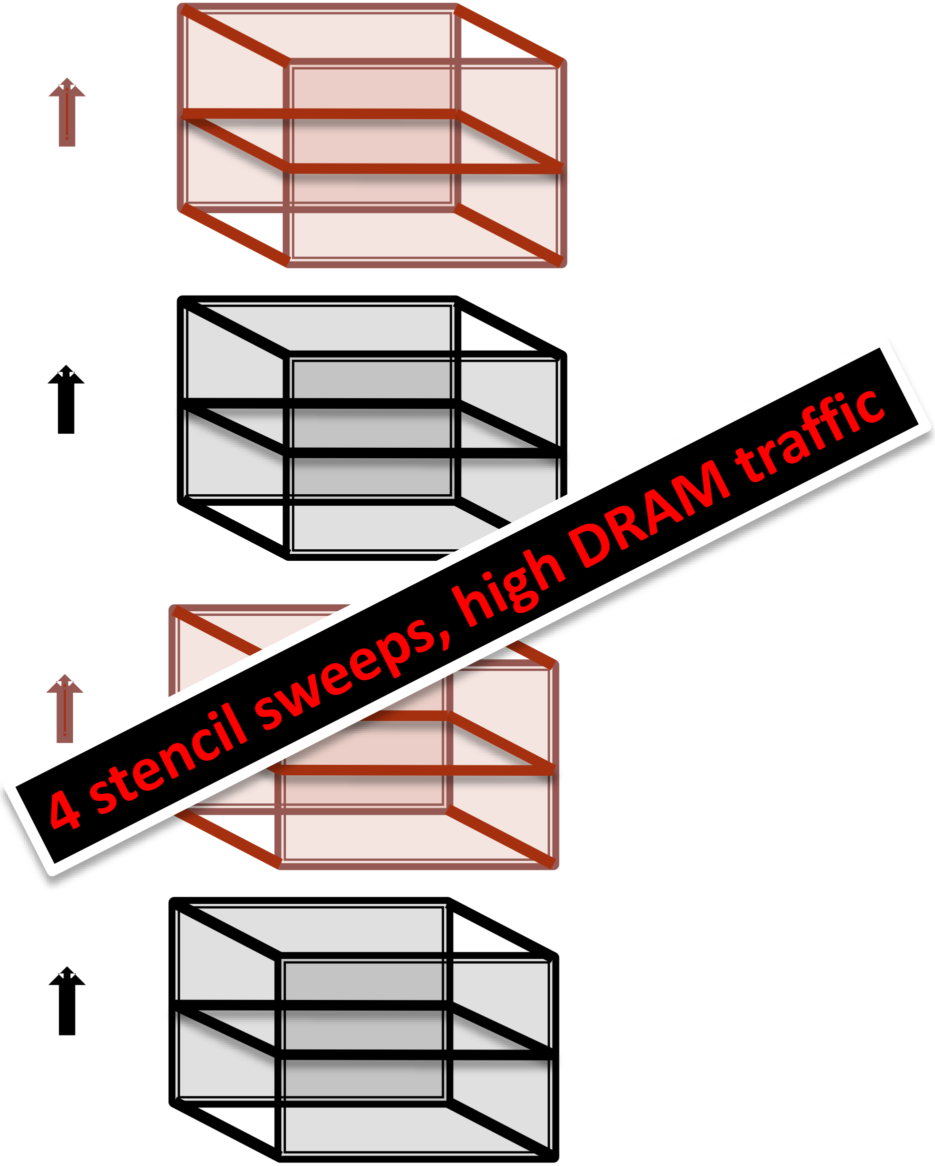
map1 := { [k,j,i] ➔ [t,k',j',i'] :
0 ≤ t < d &&
k-d+1+t ≤ k'< k+d-t &&
j-d+1+t ≤ j' < j+d-t &&
i-d+1+t ≤ I' < i+d-t};

map2 := {[color] ➔ [color+t]};

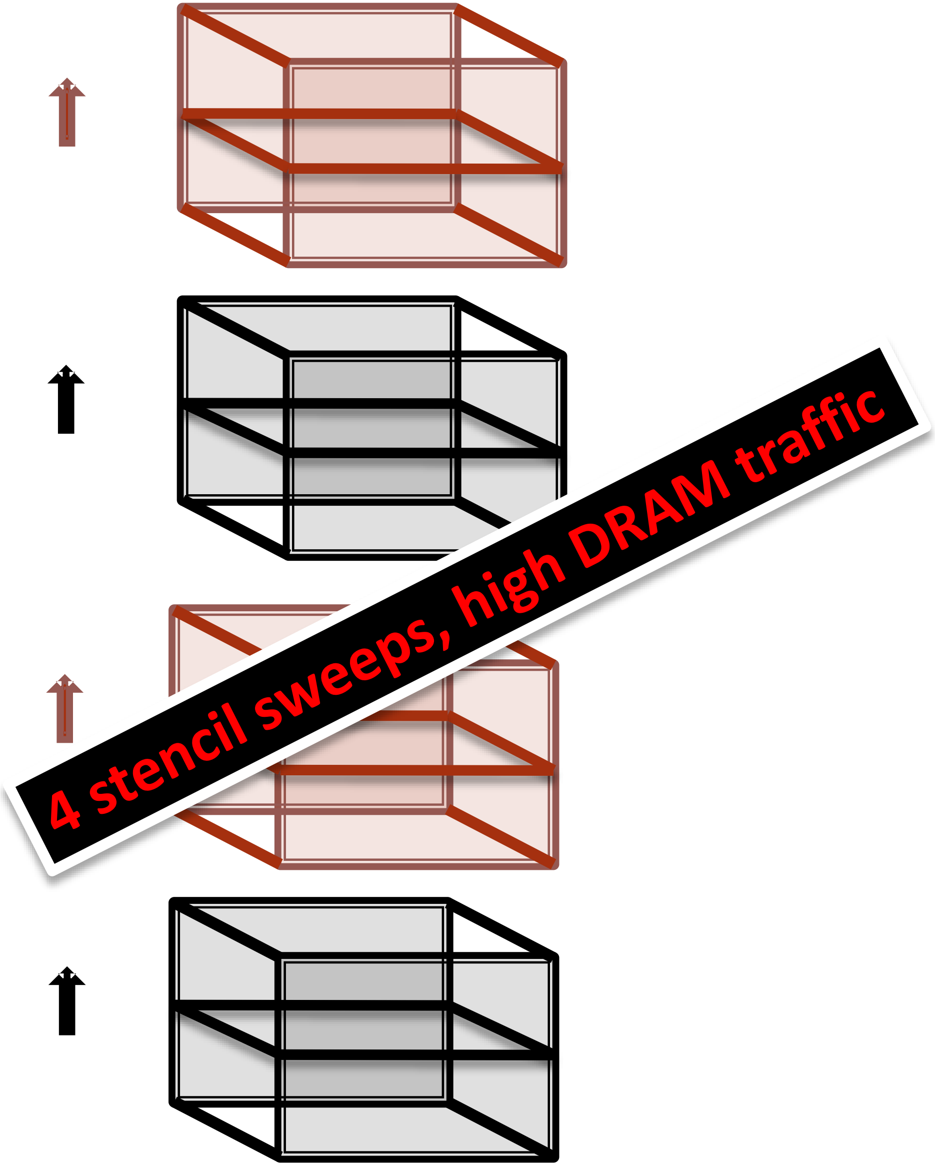
Wavefront

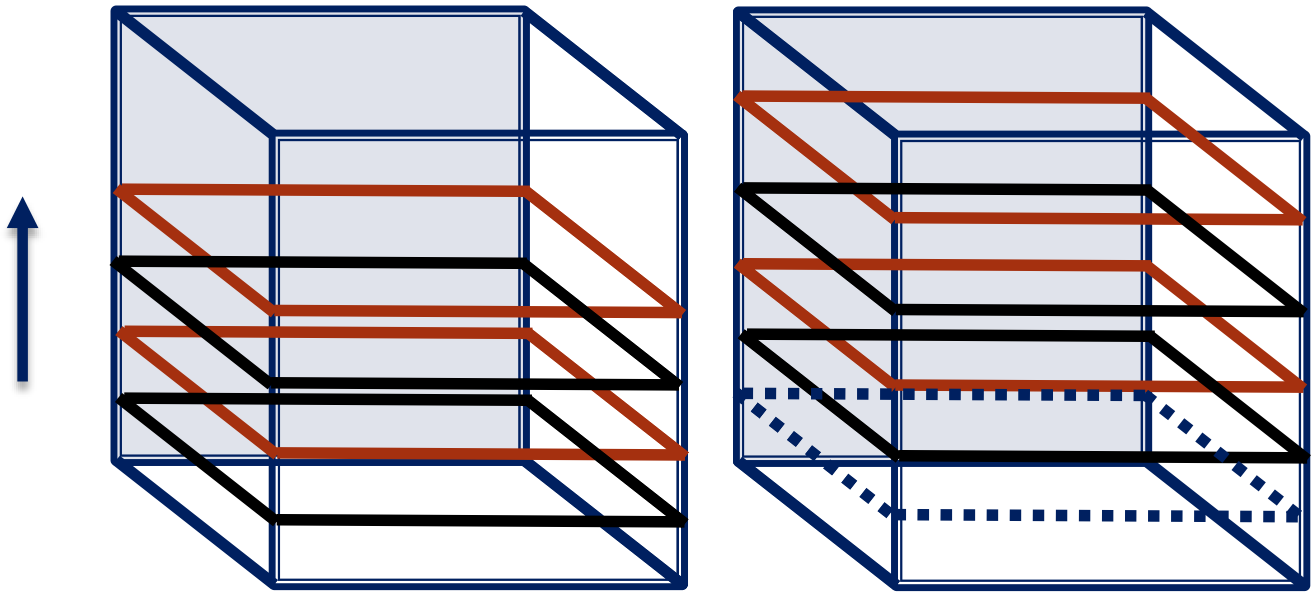


Wavefront

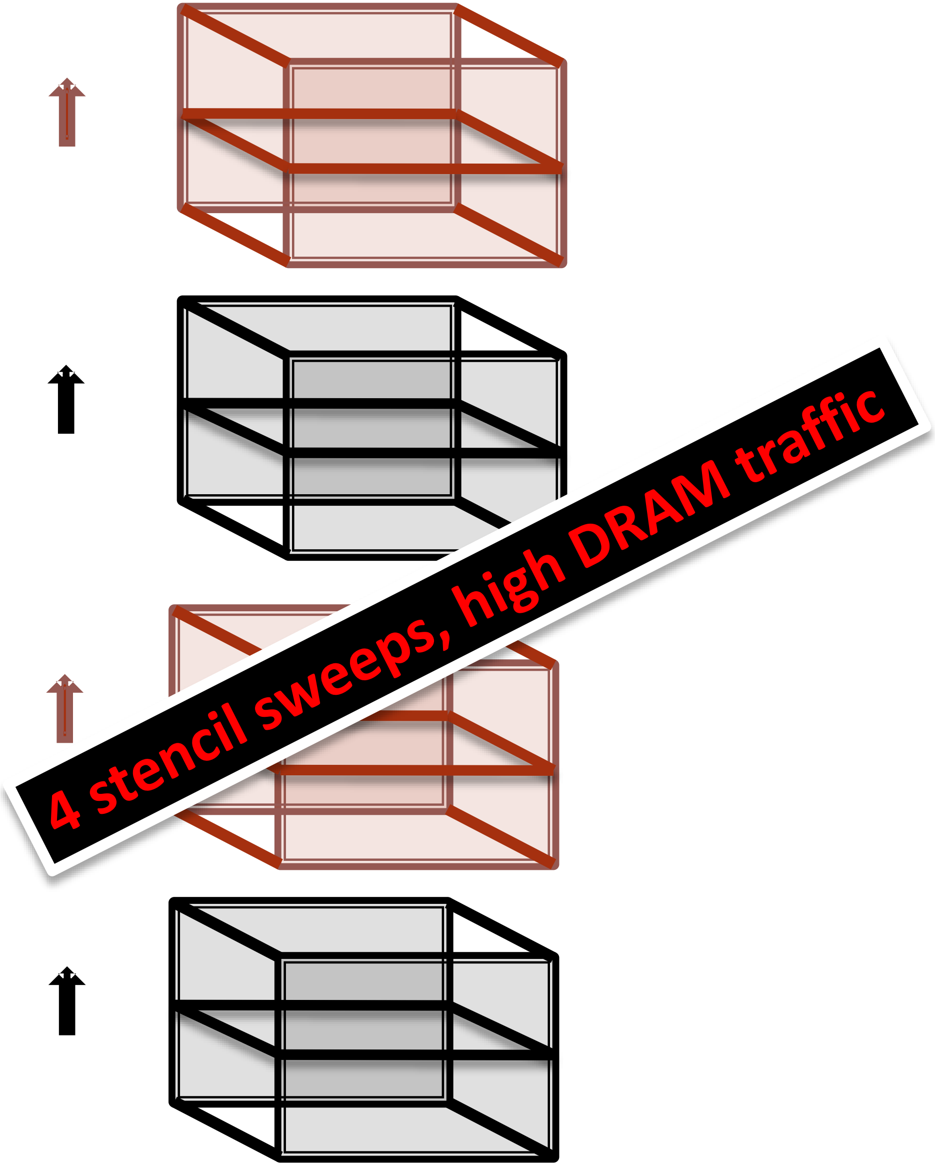


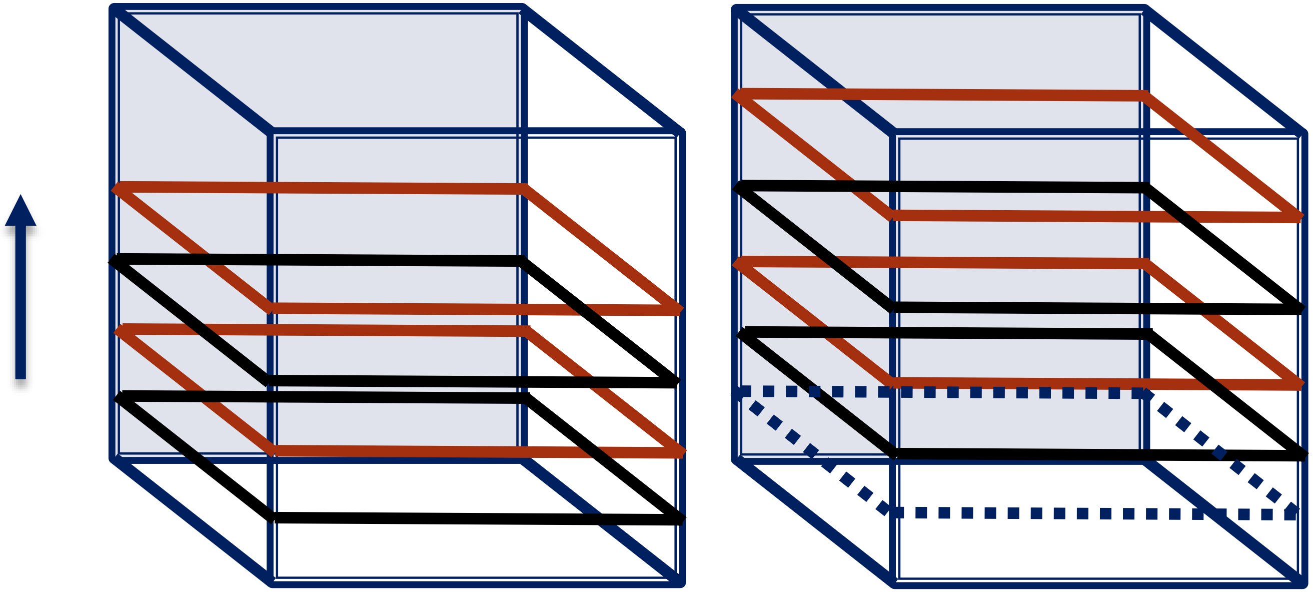
Wavefront





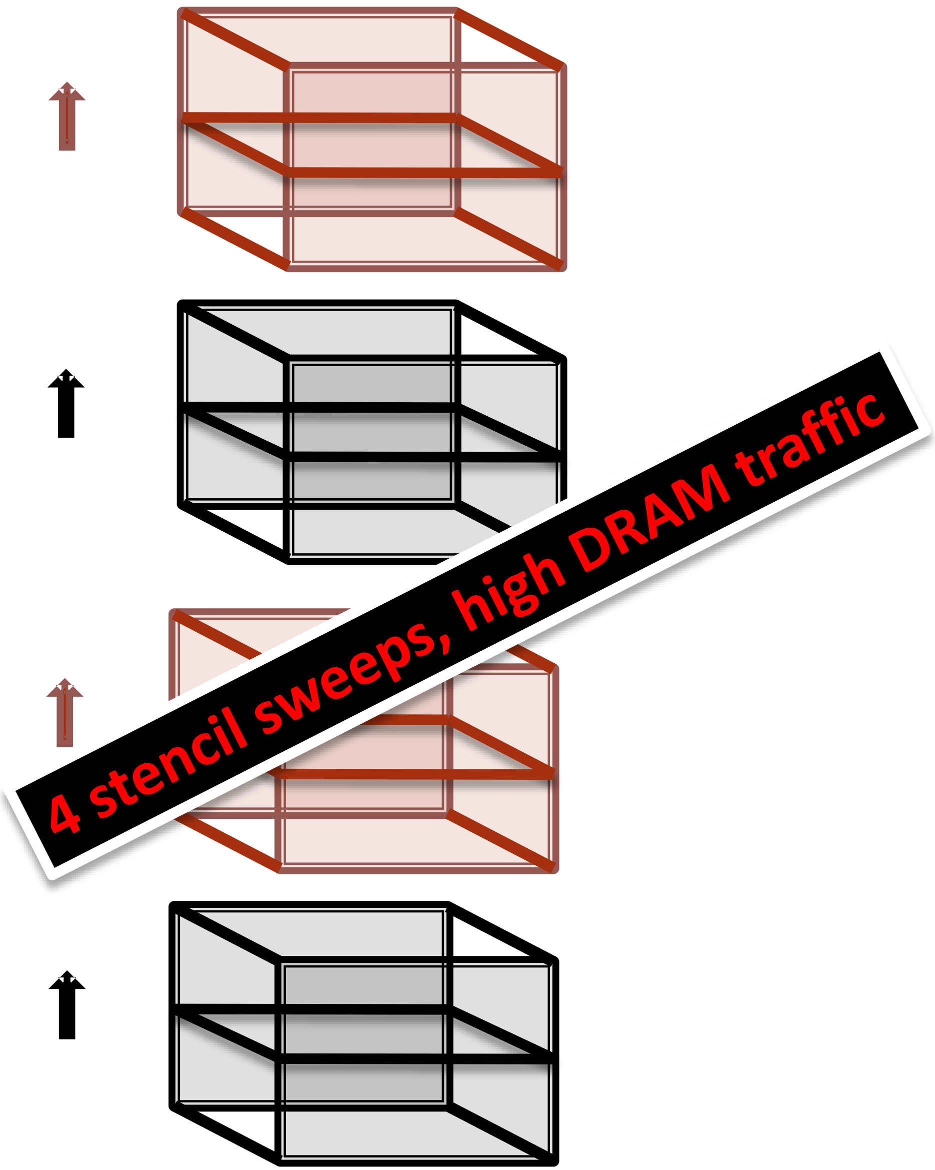
Wavefront



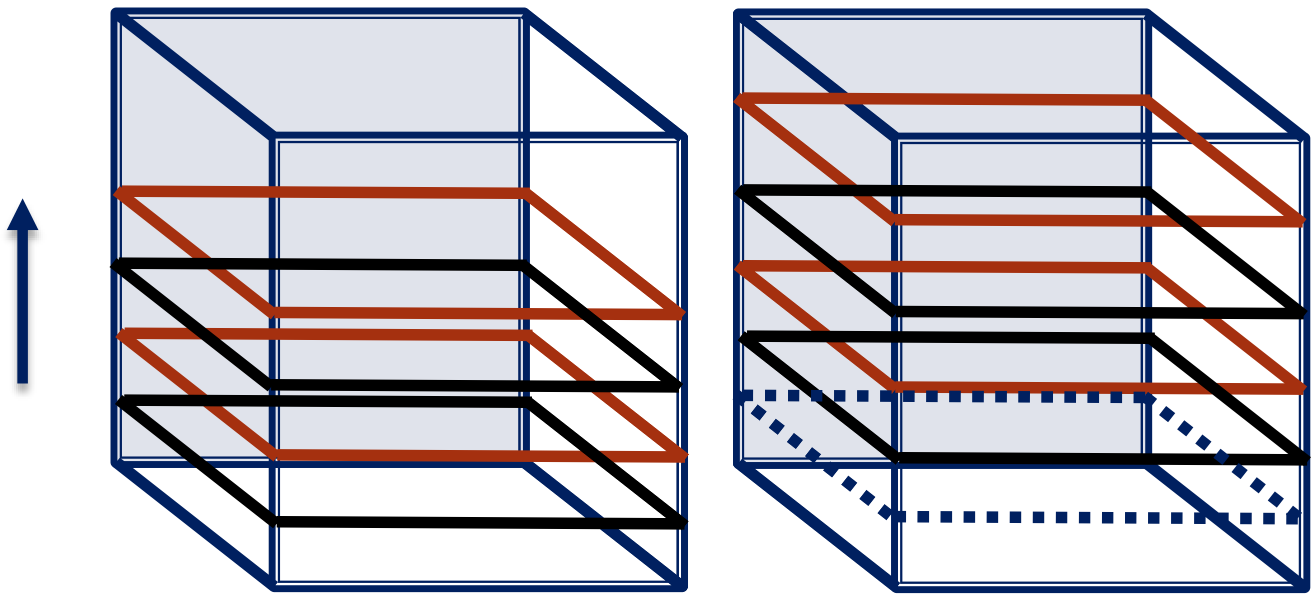


- Single stencil sweep
- Larger working set
- Thread blocking needed

Wavefront



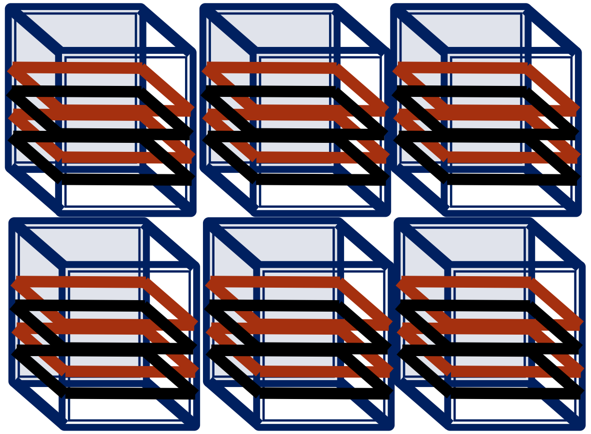
4 stencil sweeps, high DRAM traffic



- Single stencil sweep
- Larger working set
- Thread blocking needed

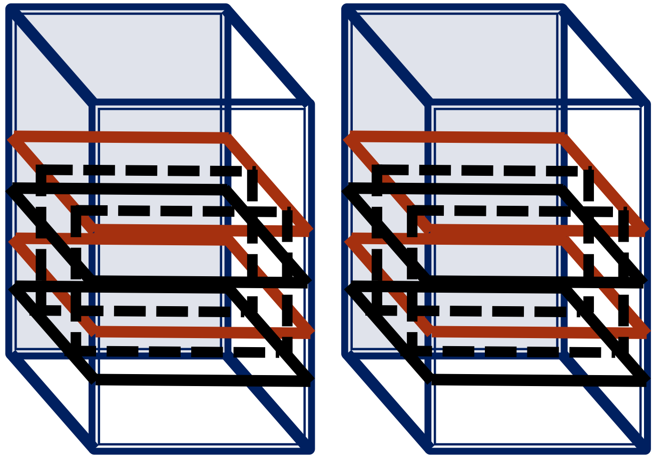
Wavefront vs. Fusion depends on box size and machine!

Inter-Box Parallelism
Thread Configuration <6,1>

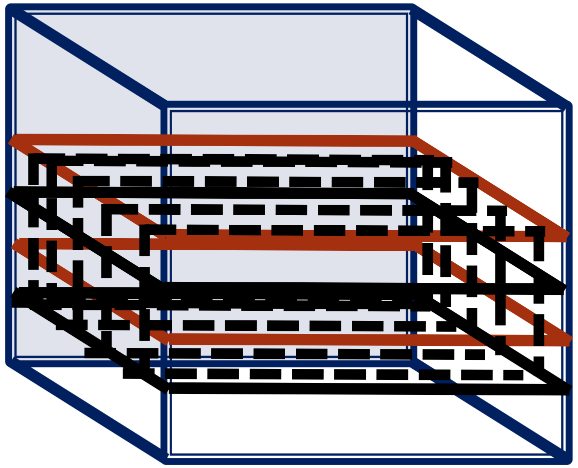


Parallel Decomposition

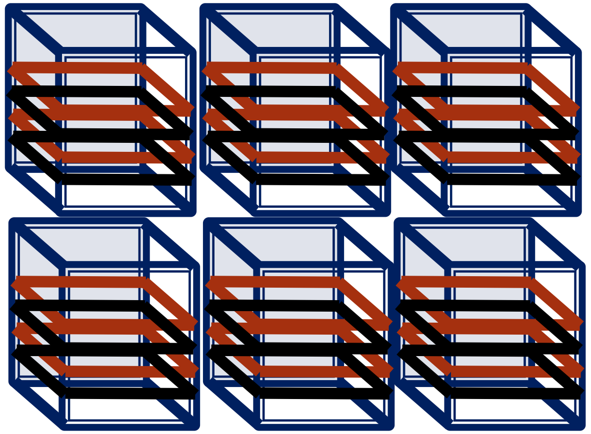
Nested Parallelism
Thread Configuration <2,3>



Intra-Box Parallelism
Thread Configuration <1,6>



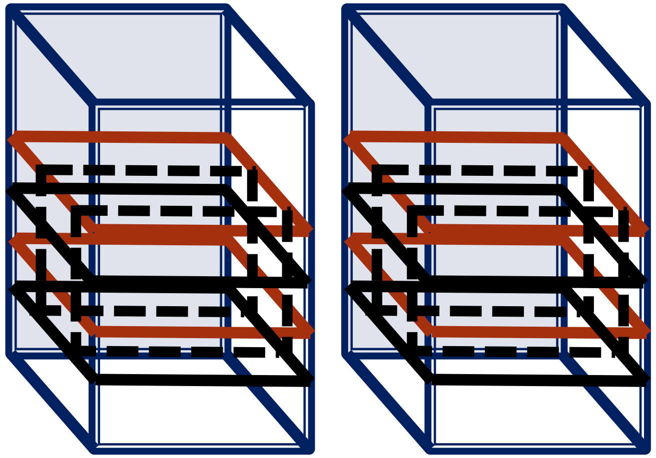
Inter-Box Parallelism
Thread Configuration <6,1>



Best parallel code generation strategy depends on box size and machine!

Parallel Decomposition

Nested Parallelism
Thread Configuration <2,3>



Intra-Box Parallelism
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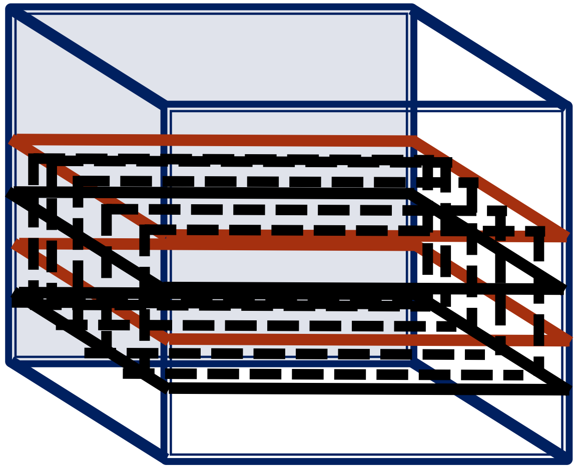

Wavefront and Parallel Code Generation

```
#pragma omp parallel private (...) num_threads(y)
{
   tid=omp_get_thread_num();
   for (k = -3; k <= 66; k++) {
       for (t = 0; t <= min(3,intFloor(t+3,2)); t++) {
          for (j = 6*tid-3; j <= min(6*tid+2,66); j++) {
              for (i= t-3+intMod(-k-color-j-(t-3),2); i<=-t+66; i+=2) {
                  S0(t,k-t,j,i); /* Laplacian */
                  S1(t,k-t,j,i); /* Helhmoltz */
                  S2(t,k-t,j,i); /* GSRB      */
               }
           }
      }
      #pragma omp barrier (or explicit locks)
   }
}
```

Wavefront = skew + permute

OpenMP parallel code generation

- The j-loop is tiled
- Each tile is assigned a thread
- Use spin locks or OMP BARRIER

Wavefront and Parallel Code Generation

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#pragma omp parallel private (...) num_threads(y)
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        for (j = 6*tid-3; j <= min(6*tid+2,66); j++) {
           for (i= t-3+intMod(-k-color-j-(t-3),2); i<=-t+66; i+=2) {
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              S1(t,k-t,j,i); /* Helhmoltz */
              S2(t,k-t,j,i); /* GSRB      */
           }
        }
     }
     #pragma omp barrier (or explicit locks)
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}
```

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        for (j = 6*tid-3; j <= min(6*tid+2,66); j++) {
           for (i= t-3+intMod(-k-color-j-(t-3),2); i<=-t+66; i+=2) {
              S0(t,k-t,j,i); /* Laplacian */
              S1(t,k-t,j,i); /* Helhmoltz */
              S2(t,k-t,j,i); /* GSRB      */
           }
        }
     }
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  }
}
```

Wavefront = skew + permute

OpenMP parallel code generation

- The j-loop is tiled
- Each tile is assigned a thread
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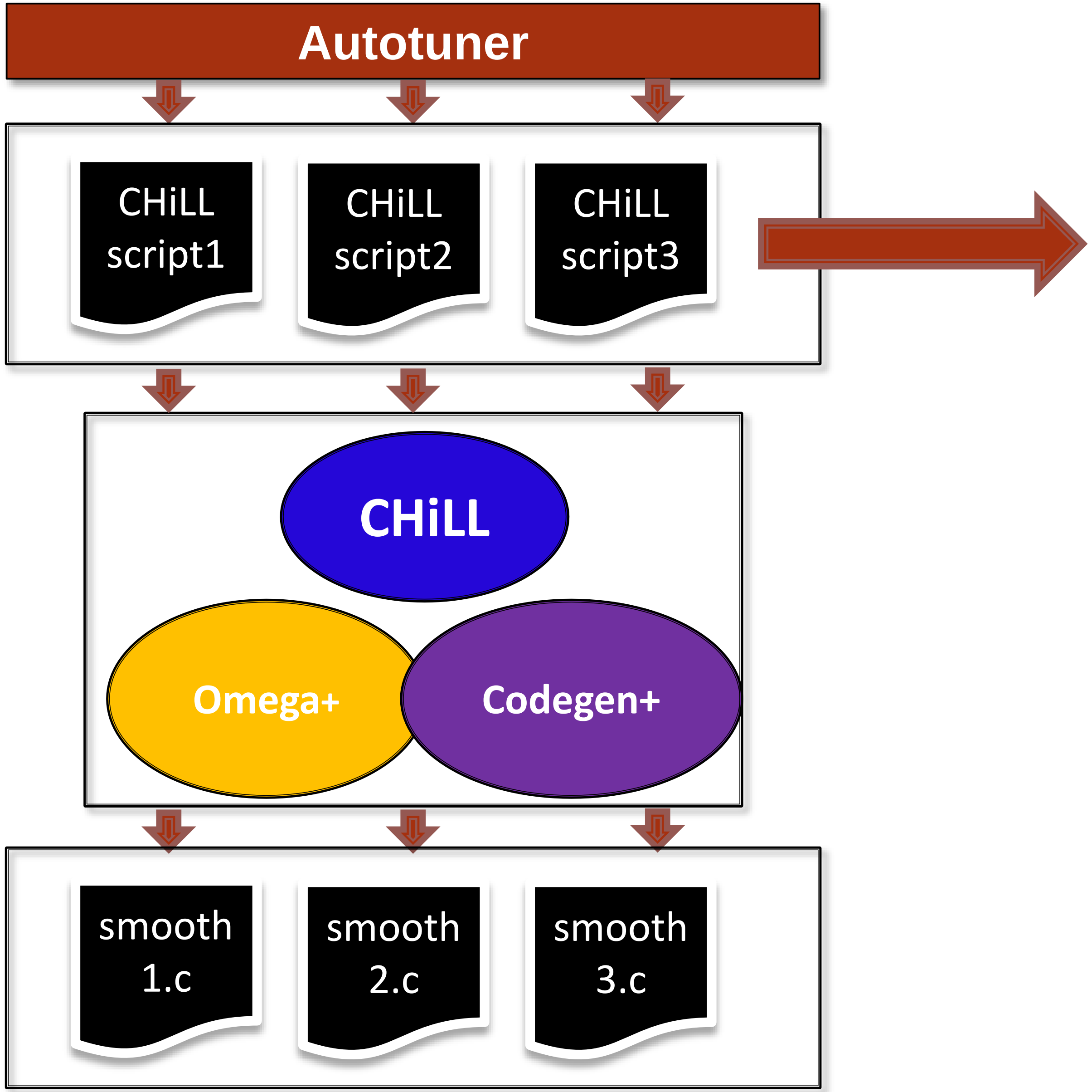
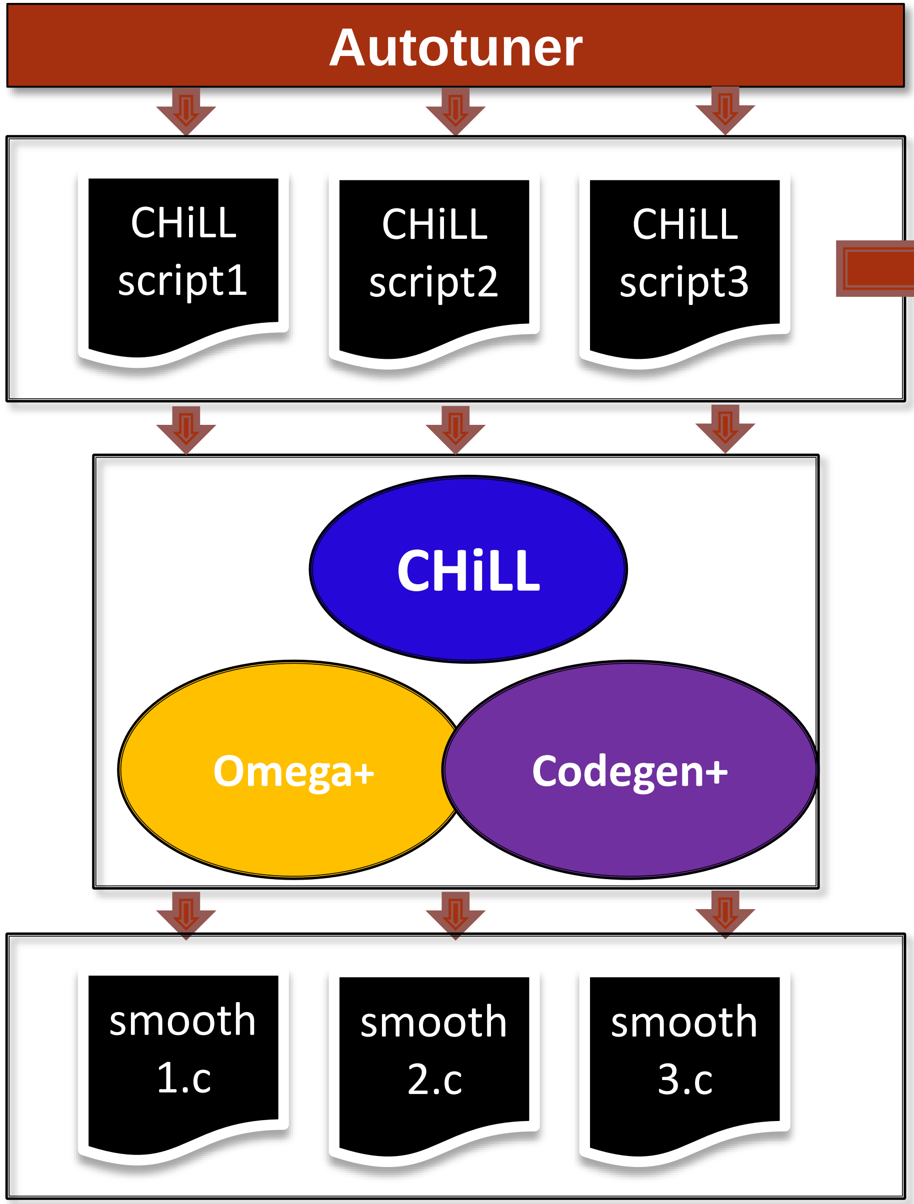
Experimental Methodology




```
known (d = 4) #d sets ghost zone
original()
skew ([0,1,2], 2, [2,1])
permute ([2,1,3,4])
tile(s0,3,TJ,2,counted)
gen_omp_parallel_region (locks, y)
```

Experimental Methodology

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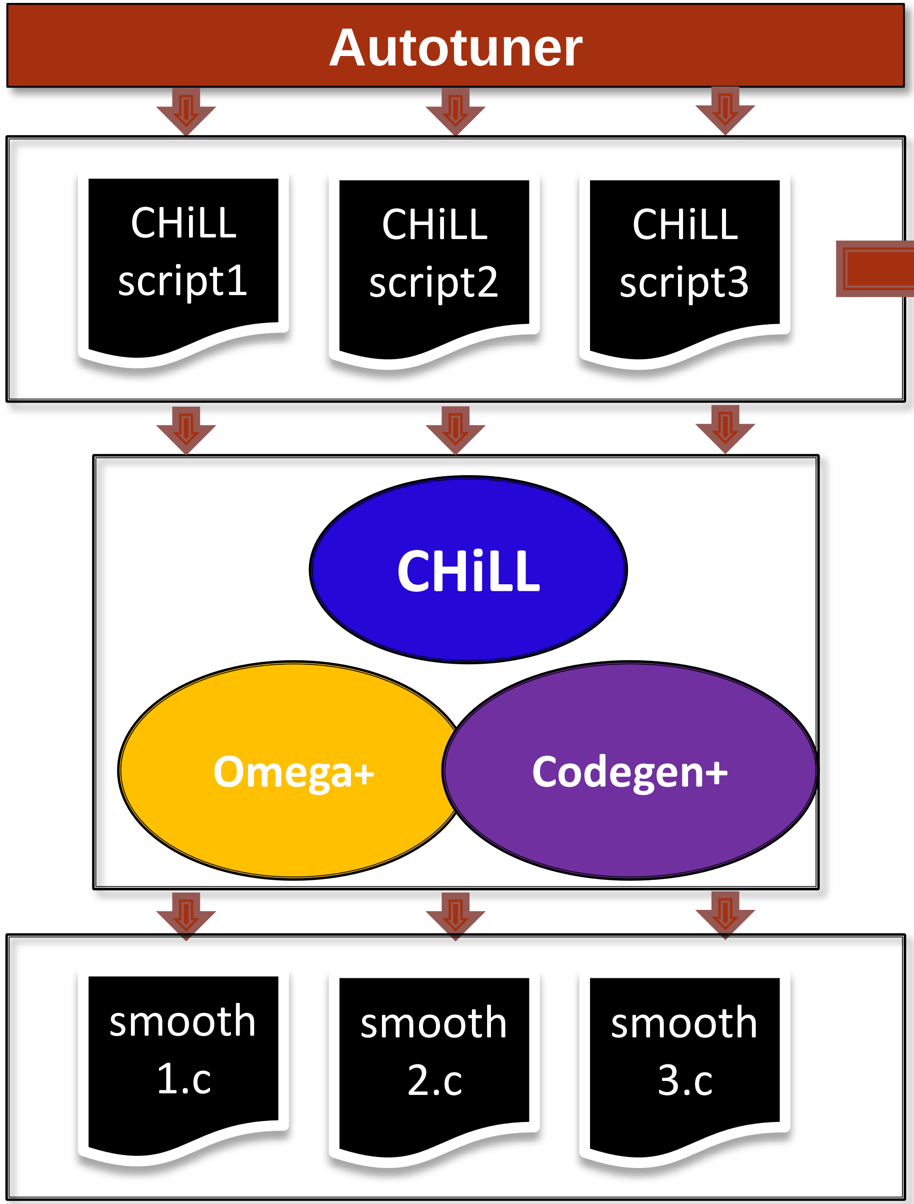
gen_omp_parallel_region (locks, y)
```

Sets ghost zone depth

Experimental Methodology

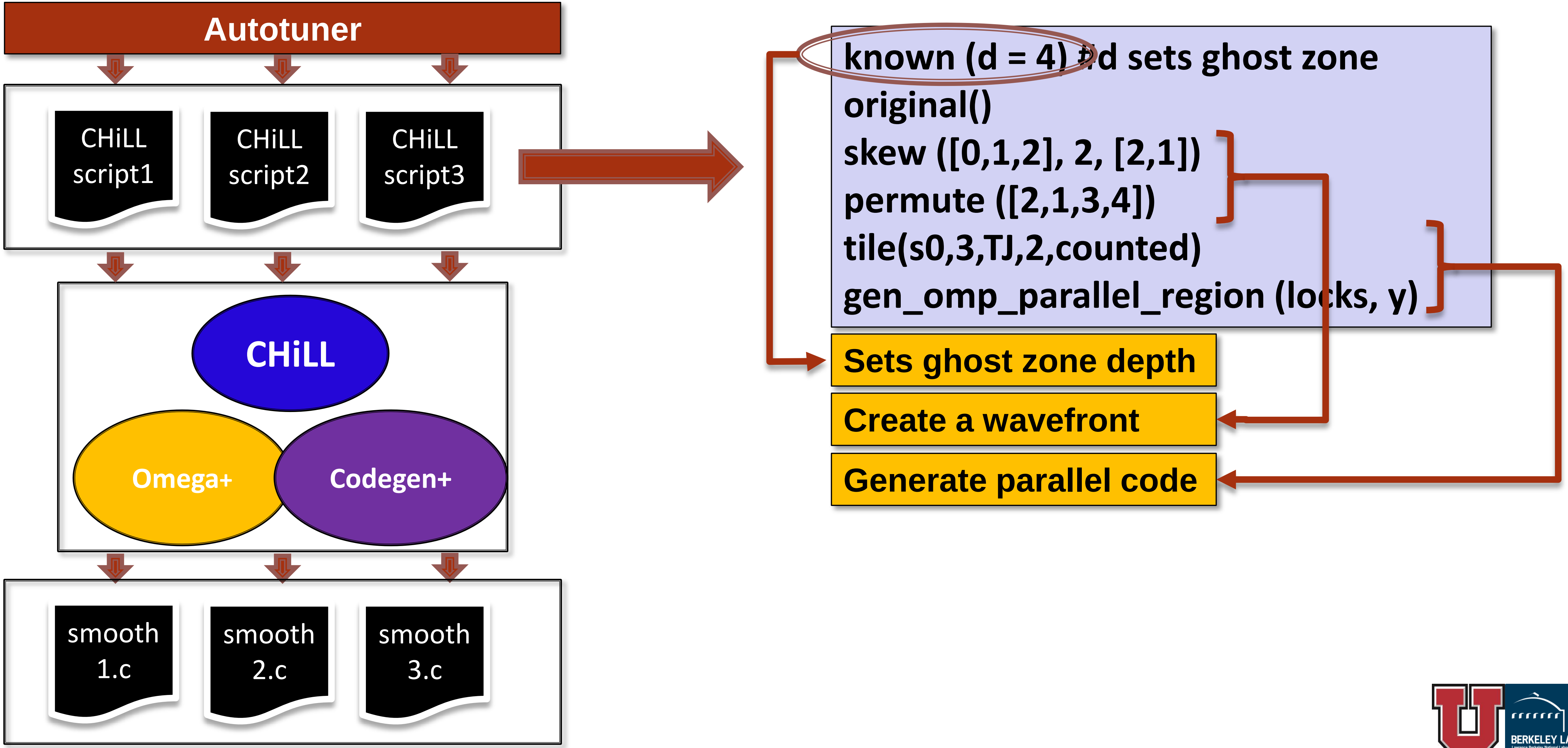



```
known (d = 4) #d sets ghost zone
original()
skew ([0,1,2], 2, [2,1])
permute ([2,1,3,4])
tile(s0,3,TJ,2,counted)
gen_omp_parallel_region (locks, y)
```

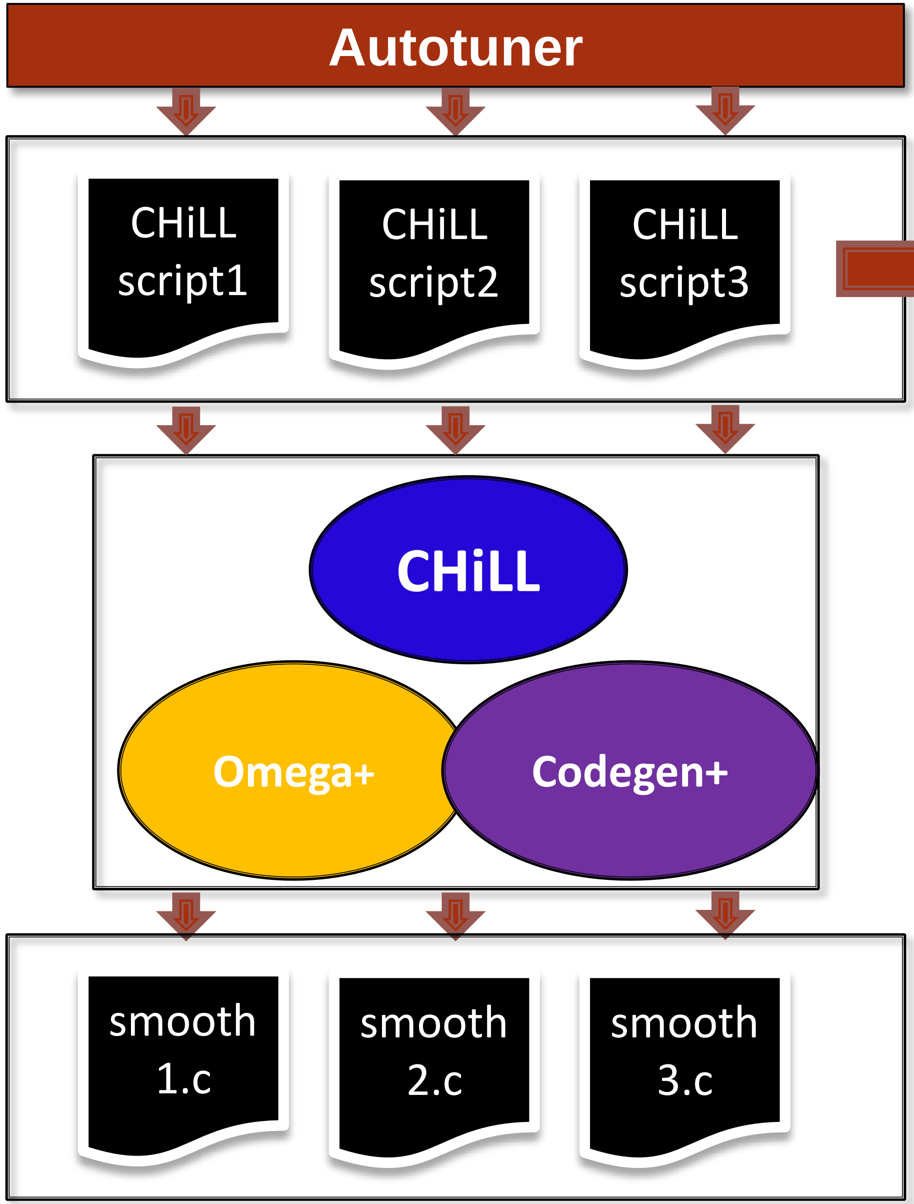
Sets ghost zone depth

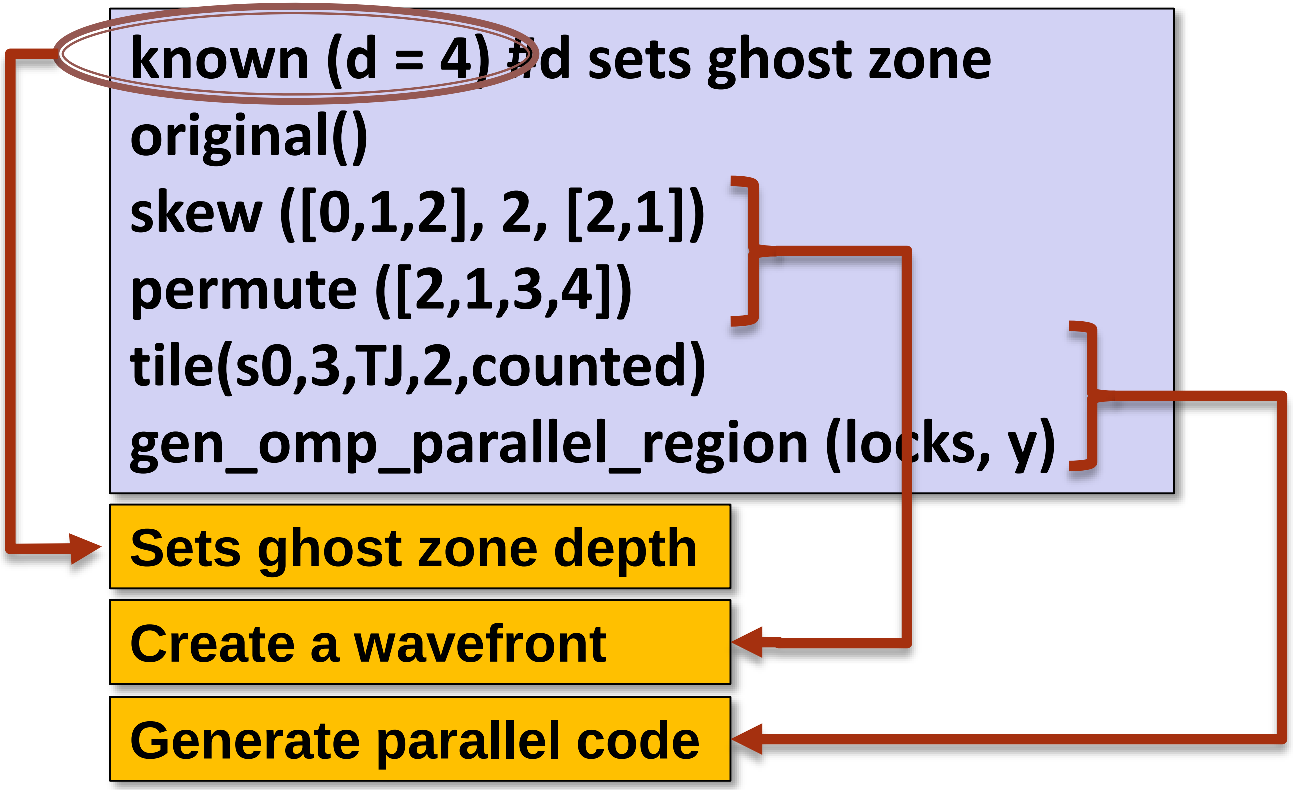
Create a wavefront

Experimental Methodology



Experimental Methodology





For each level of the V-cycle, our search space is:

- Ghost zone depth ‘d’ {1,2,4}
- Fused Code or Wavefront variant
- Thread decomposition <x, y>, where x*y = #cores/chip

Experimental Methodology

Problem configuration

- 256^3 problem size (domain), decomposed to 64^3 boxes
- Variable coefficient 3D 7-point Gauss-Seidel red-black smooth operator
- Periodic boundary conditions
- V-cycle run 10 times

Target Architectures

Code was run on a single node on two NERSC machines. Hopper, a Cray XE6 and the new Cray XC30 Edison Phase II.

Hopper	Edison (Phase II)
AMD Opteron Cores	Intel Sandy Bridge Cores
4 chips per node, 6 cores per chip	2 chips per node, 12 cores per chip

Experimental Methodology

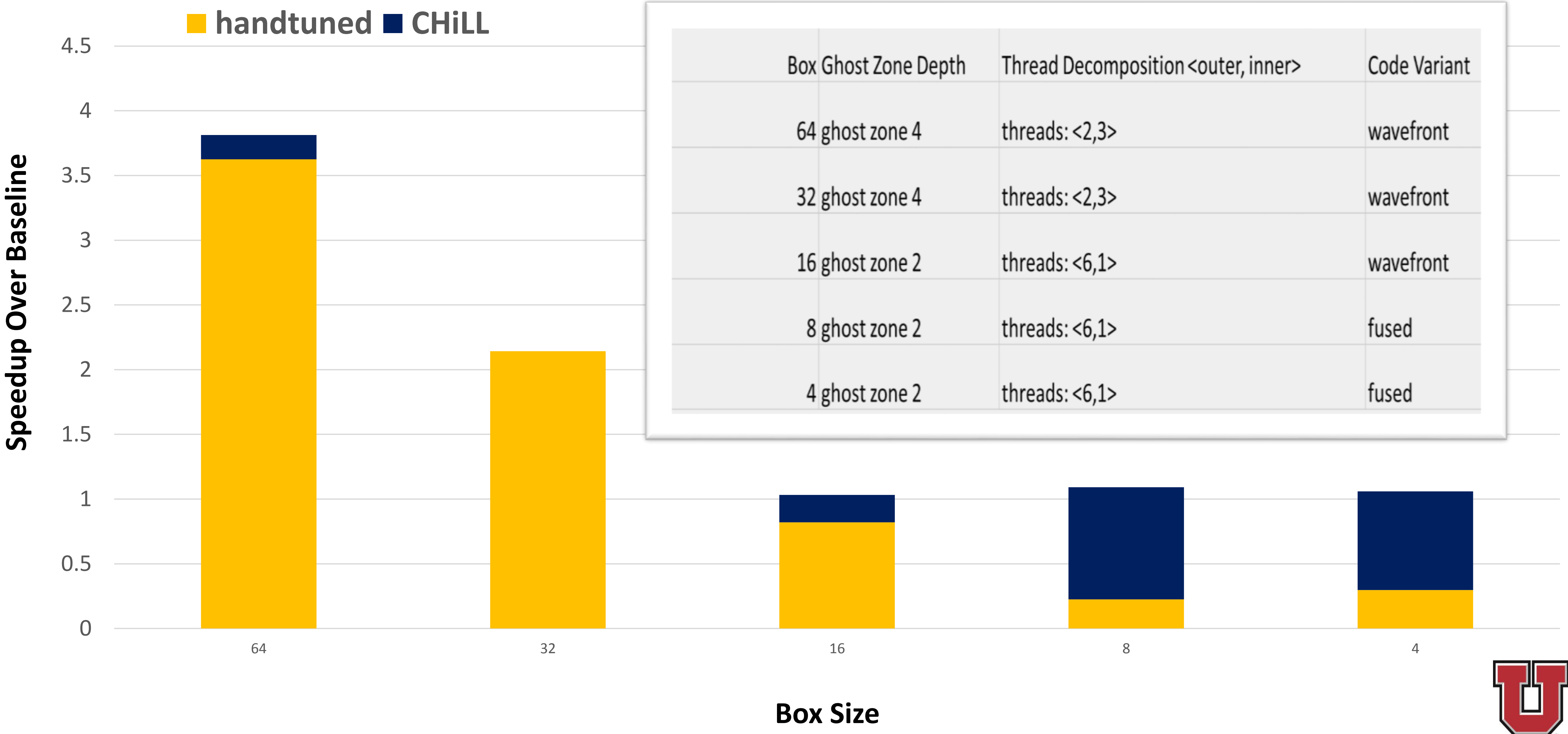
Baseline Code

- Operators (Laplacian, Helmholtz, GSRB) not fused
- Ghost zone depth is one
- Inter-box thread decomposition

Hand tuned Code : miniGMG

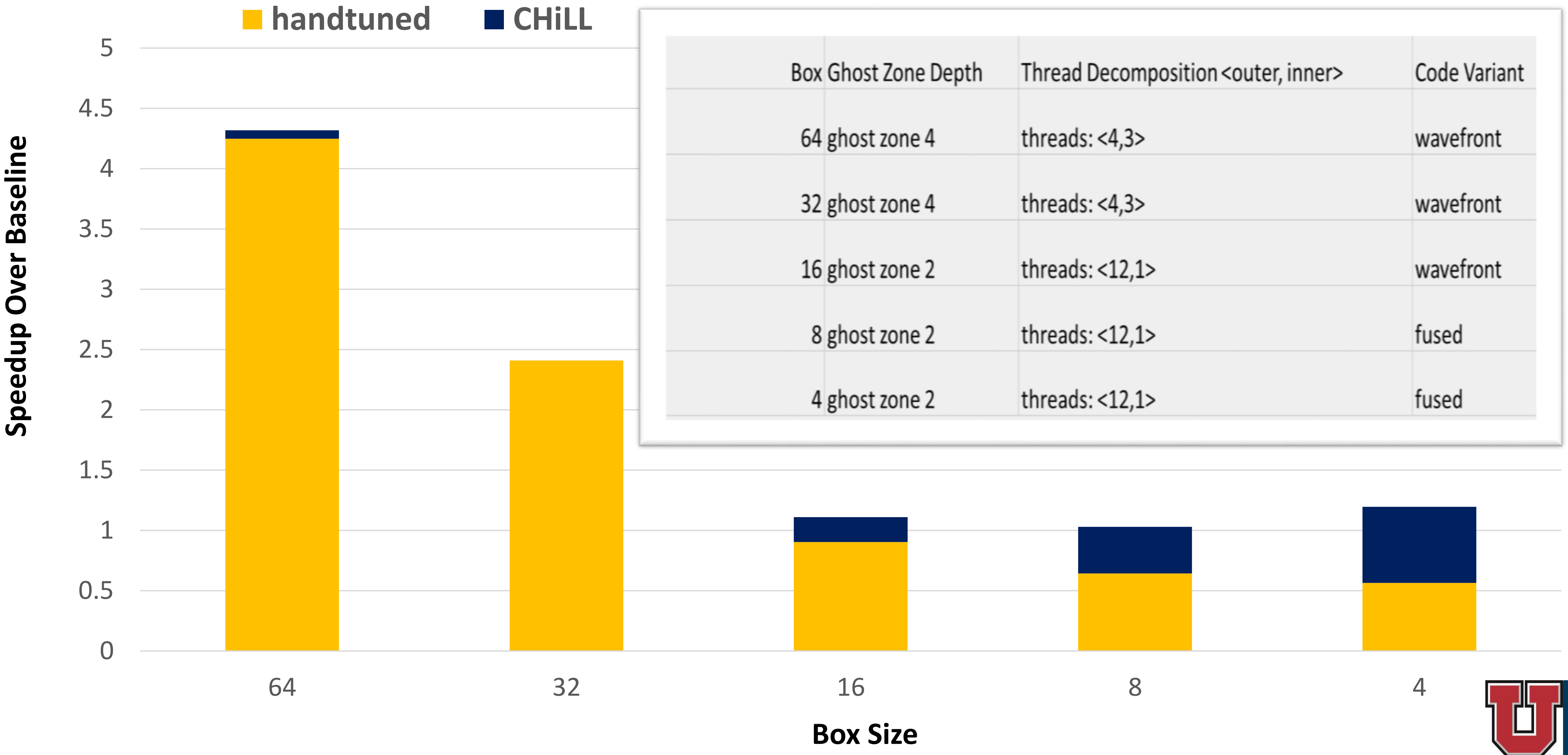
High performance code from Samuel Williams et al. Supercomputing’12 paper : “Optimization of Geometric Multigrid for Emerging Multi- and Many core Processors”

Performance of Smooth on Hopper



Box	Ghost Zone Depth	Thread Decomposition <outer, inner>	Code Variant
64	ghost zone 4	threads: <2,3>	wavefront
32	ghost zone 4	threads: <2,3>	wavefront
16	ghost zone 2	threads: <6,1>	wavefront
8	ghost zone 2	threads: <6,1>	fused
4	ghost zone 2	threads: <6,1>	fused

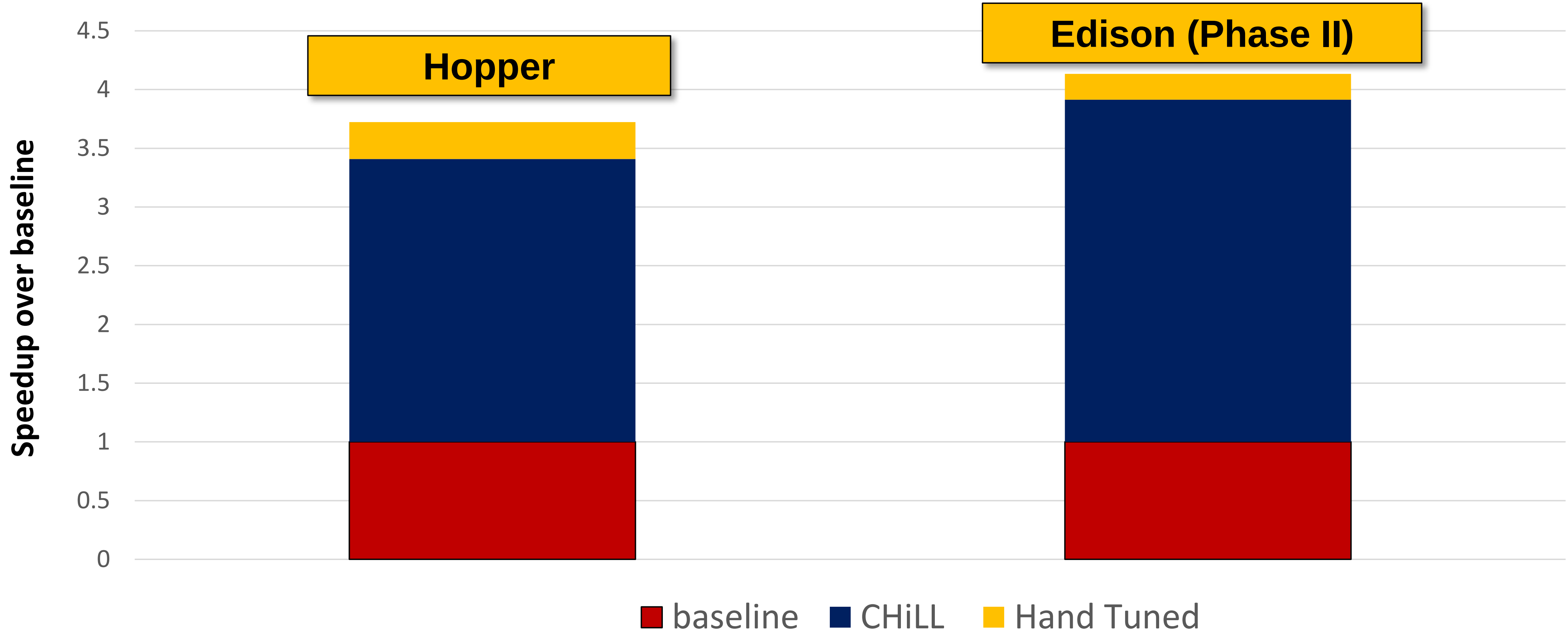
Performance of Smooth on Edison (Phase II)



Box	Ghost Zone Depth	Thread Decomposition <outer, inner>	Code Variant
64	ghost zone 4	threads: <4,3>	wavefront
32	ghost zone 4	threads: <4,3>	wavefront
16	ghost zone 2	threads: <12,1>	wavefront
8	ghost zone 2	threads: <12,1>	fused
4	ghost zone 2	threads: <12,1>	fused

Overall Speedup for Box Size 64

Total Time = Smooth + Residual + Restriction



Summary and Conclusion

Need For Autotuning And Compiler Support

At each level of the V-cycle we *need* to autotune for :

- Ghost zone depth
- Create a wavefront computation or use simple fused loops
- Thread decomposition

Higher Performance From Searching a Rich Space of Variants

Generated code betters hand tuned code without

- Software prefetching
- SSE/AVX code

Compiler-generated code variant has different threading configuration than manually tuned code!

Questions?

Extra

We use a double-precision, finite volume discretization of the variable-coefficient operator $L = a\vec{\alpha}I - b\nabla\vec{\beta}\nabla$ with periodic boundary conditions as the linear operator within our test problem. Variable-coefficient is an essential (yet particularly challenging) facet as most real-world applications demand it. The right-hand side (f) is $sin(2\pi x)sin(2\pi y)sin(2\pi z)$ on the [0,1] cubical domain. The u, f, and $\vec{\alpha}$ are cell-centered data, while the $\vec{\beta}$'s are face-centered.