

Physical Oscillator Model for Supercomputing

Ayesha Afzal, Georg Hager, Gerhard Wellein

*Erlangen National High Performance Computing Center (NHR@FAU)
Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany*



Coupled Oscillators: Fireflies and Metronomes!



John Bernard Estrada, YouTube <https://youtu.be/2IVBDDVPI68?si=vaEn-Ki4sAZHQsb->



UCLA Physics, YouTube <https://youtu.be/T58IGKREubo?si=LGaB-xWwhiMI0uLg>

Coupled Oscillators: The Kuramoto Model

The diagram illustrates the Kuramoto model equation for the time derivative of the phase of oscillator i , $\dot{\theta}_i$. The equation is
$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i)$$
 Five teal boxes with white text are connected to parts of the equation by thin teal lines: 'Actual frequency' points to $\dot{\theta}_i$; 'Natural (base) frequency' points to ω_i ; 'Coupling strength' points to K ; 'All-to-all coupling' points to the summation symbol $\sum$; and 'Interaction potential' points to the sine function $\sin(\theta_j - \theta_i)$.

Actual frequency

Natural (base) frequency

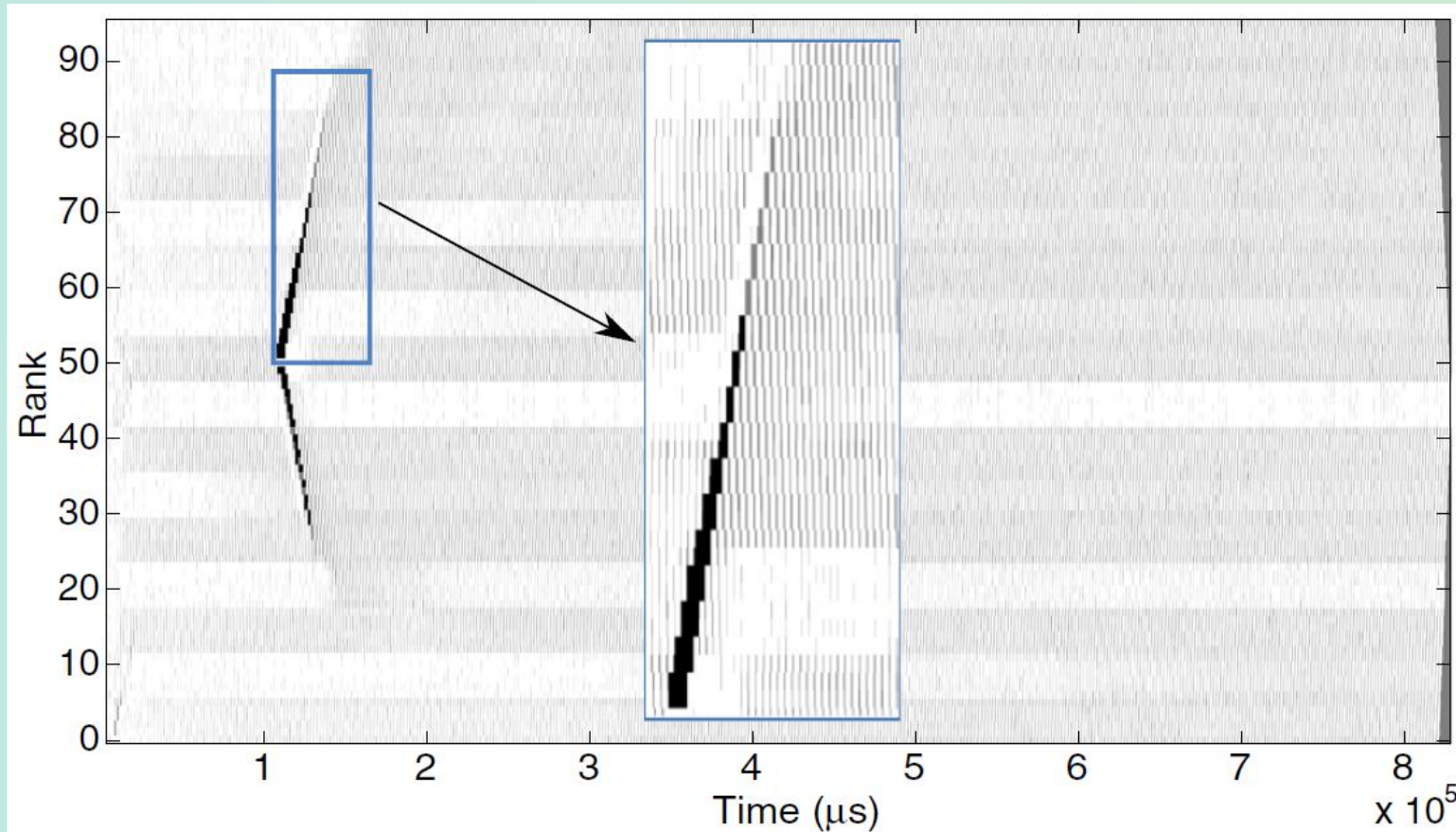
Coupling strength

All-to-all coupling

Interaction potential

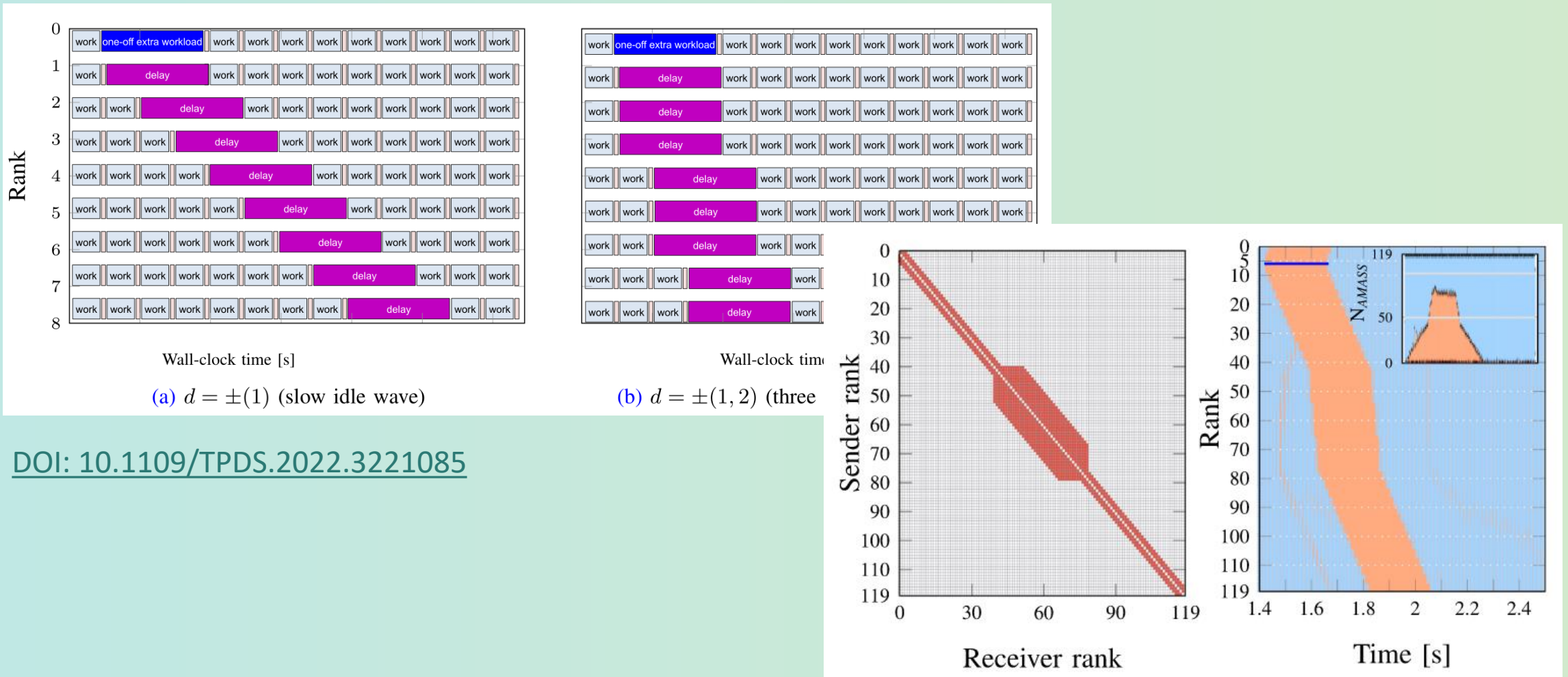
$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i)$$

Parallel Programs: Idle Wave Propagation and Decay



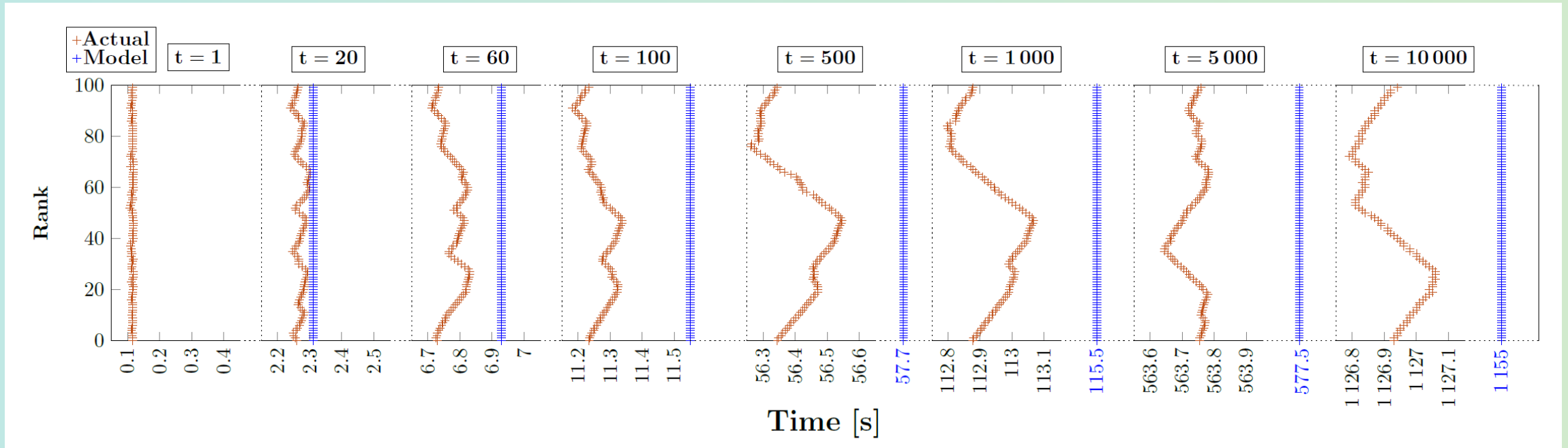
S. Markidis et al.: *Idle waves in high-performance computing*. Phys. Rev. E **91**(1), 013306 (2015). DOI: [10.1103/PhysRevE.91.013306](https://doi.org/10.1103/PhysRevE.91.013306)

Idle Wave Speed and Communication Topology



[DOI: 10.1109/TPDS.2022.3221085](https://doi.org/10.1109/TPDS.2022.3221085)

Bottlenecks, Computational Wavefronts, and Spontaneous Asynchronicity



[DOI: 10.1109/CLUSTER.2019.8890995](https://doi.org/10.1109/CLUSTER.2019.8890995)

Core Analogies

MPI processes	Coupled oscillators
Process	Harmonic oscillator
Regular compute-communicate phases	Intrinsic oscillation
Load imbalance	Spread of natural frequencies
Noise	Variation of frequencies and/or phases
Delay propagation speed	Coupling strength
Communication topology	Coupling topology

An Oscillator Model for Communicating Processes

$$\dot{\theta}_i = \omega_i + \zeta_i(t) + \frac{K}{N} \sum_j T_{ij} V(\theta_j, \theta_i, \tau_{ij}(t))$$

Frequency noise

Phase noise

Communication
topology

$$\omega_i = \frac{2\pi}{t^{comp} + t^{comm}}$$

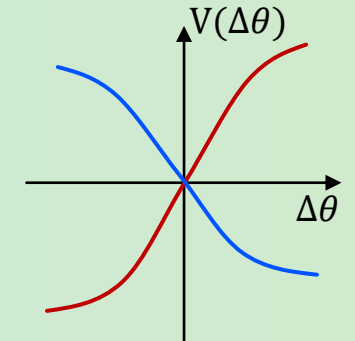
$$K = \frac{\beta\kappa}{t^{comp} + t^{comm}}$$

Requirements

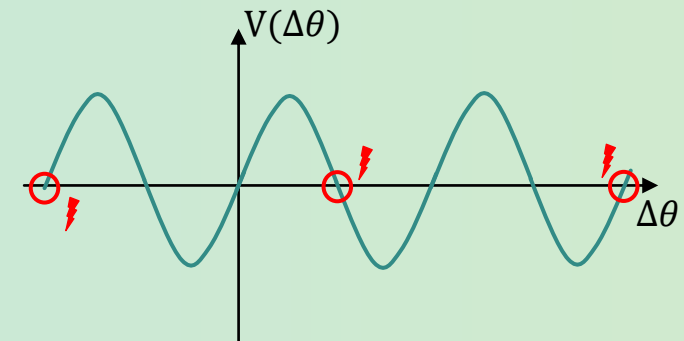
- T_{ij} must reflect the actual communication topology of the program

$$\begin{bmatrix} 1 & 0 & \dots & 1 \\ 1 & 1 & & 0 \\ 1 & 1 & \ddots & \vdots \\ \vdots & & \ddots & 1 \\ 1 & \dots & 1 & 1 \end{bmatrix}$$

- $V(\cdot)$ must allow for **re-** or **des**synchronization: $V'(0) \geq 0$



- $V(\cdot)$ must not be periodic (no phase slips)



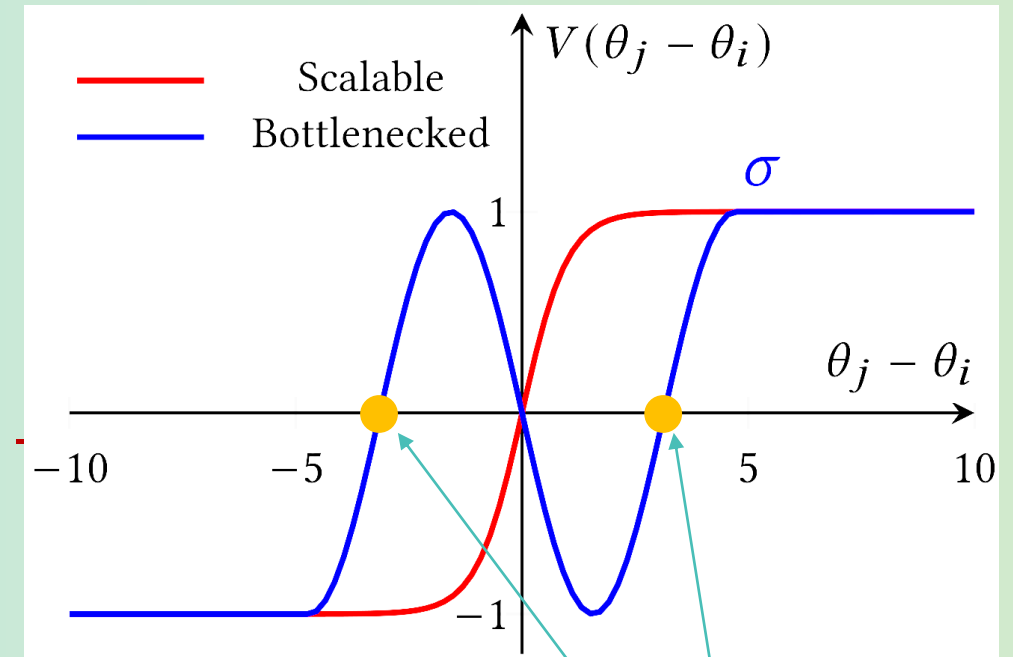
Possible Choices for $V(\cdot)$

- Resynchronizing (scalable code)

$$V(\theta_j - \theta_i) = \tanh(\theta_j - \theta_i)$$

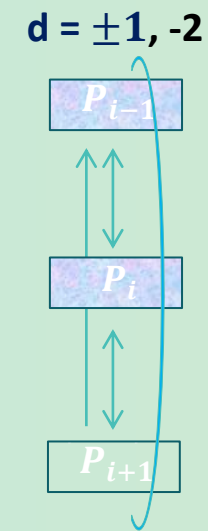
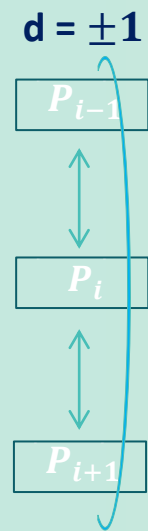
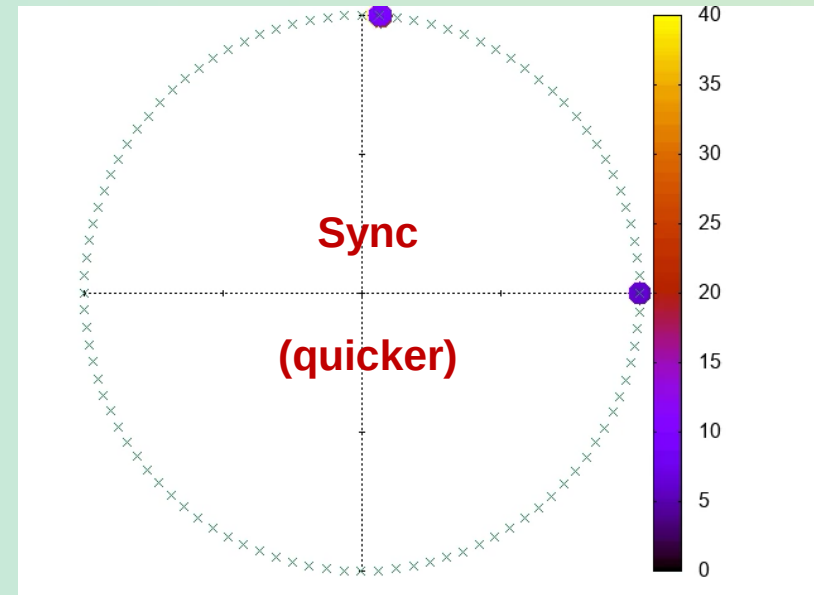
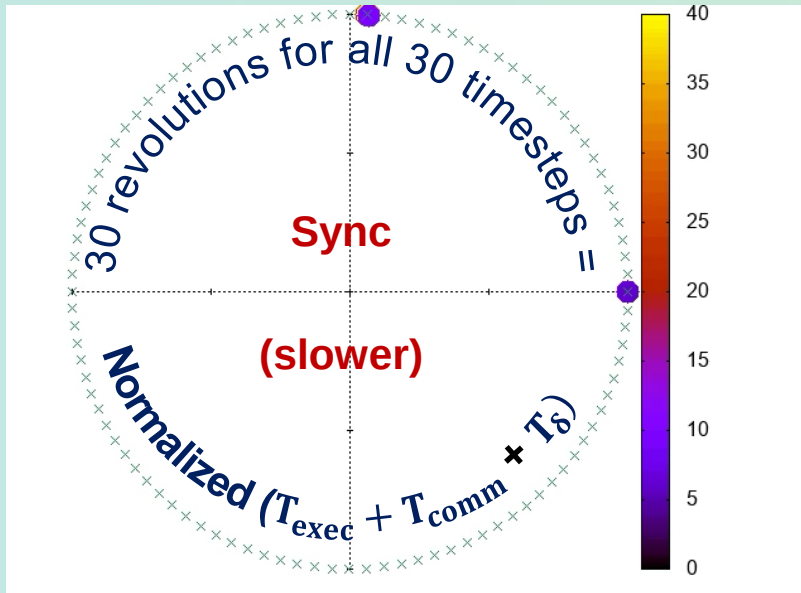
- Desynchronizing (bottlenecked code)

$$V(\theta_j - \theta_i) = \begin{cases} -\sin\left(\frac{3\pi}{2\sigma} \cdot (\theta_j - \theta_i)\right) & \text{if } |\theta_j - \theta_i| < \sigma \\ \text{sgn}(\theta_j - \theta_i) & \text{else} \end{cases}$$

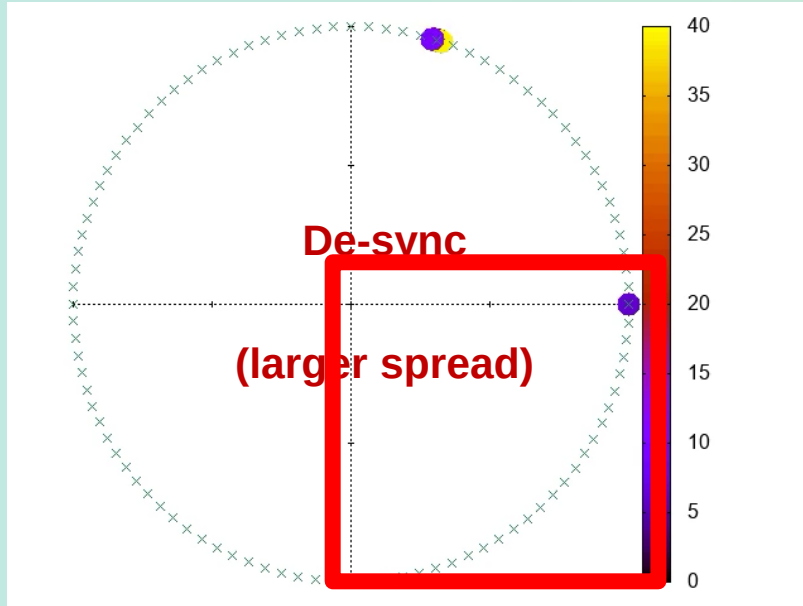


Interaction horizon
"Attractor" at $2\sigma/3$

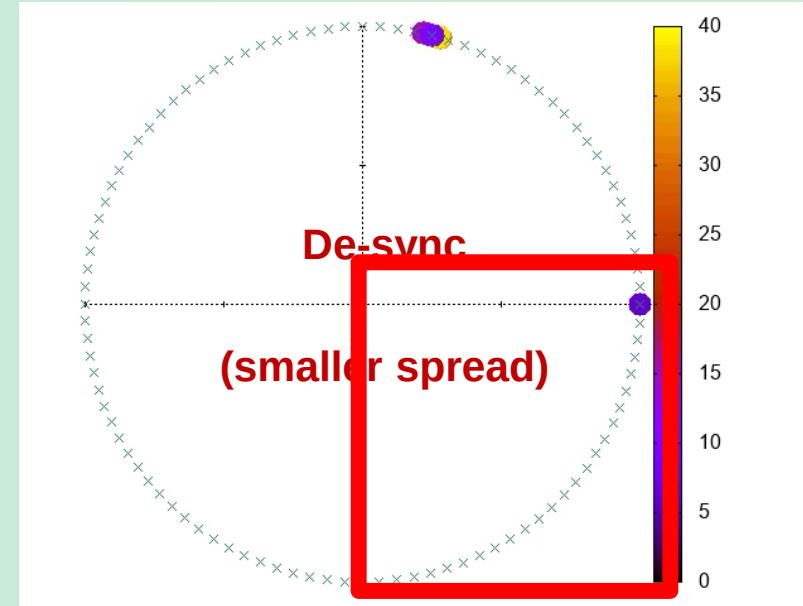
Evaluation: Scalable Code



Evaluation: Bottlenecked Code



$d = \pm 1$



$d = \pm 1, -2$

Outlook and future work

- This is a wild concept; we have no idea if it has merit
- The model provides a good qualitative representation of some of the features of parallel programs
- The model can be used to characterize systems with “interesting” collective behavior
- Co-design?
- A continuum limit is as yet unknown
- Future work
 - Quantitative exploration of the role of the noise functions
 - Connecting the symmetry-breaking transition to the Goldstone mode



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OMI4papps

Title

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Contact

Ayesha Afzal
ayesha.afzal@fau.de



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